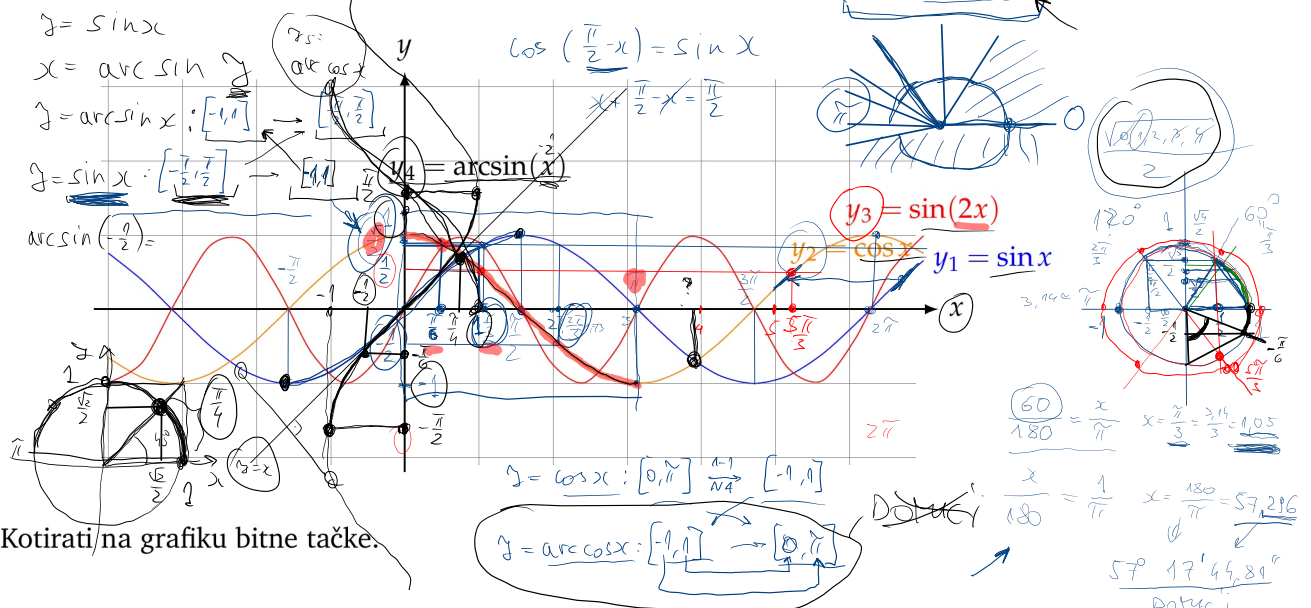
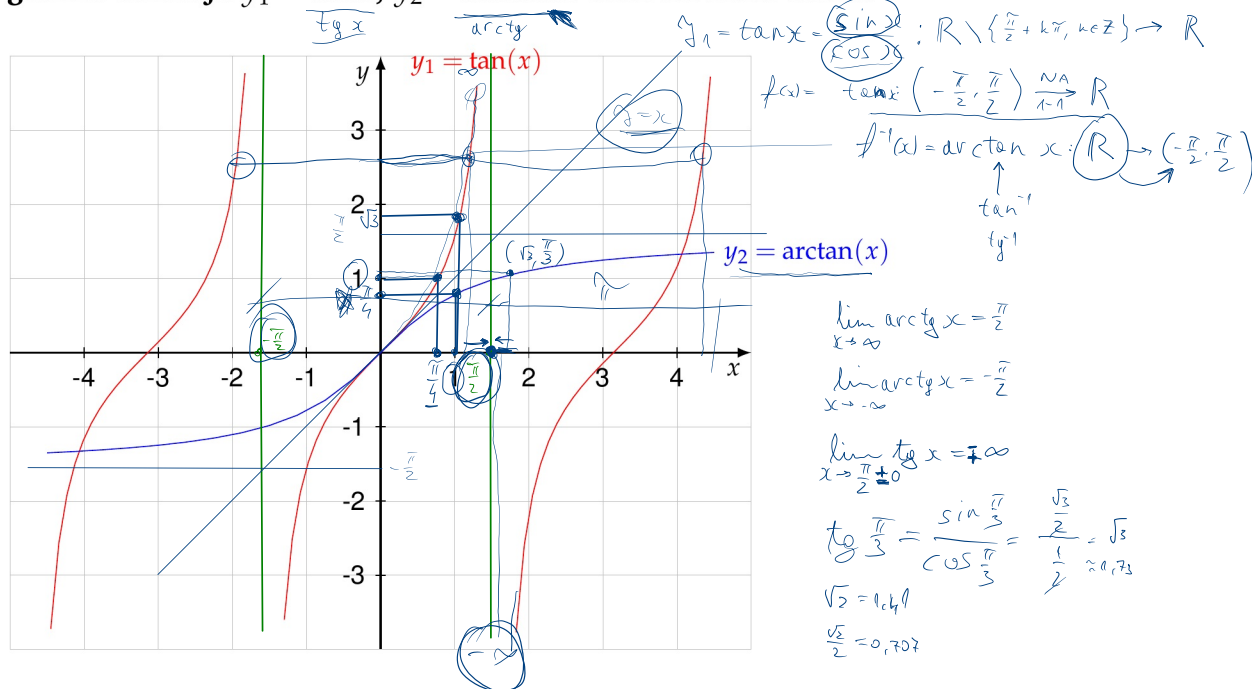


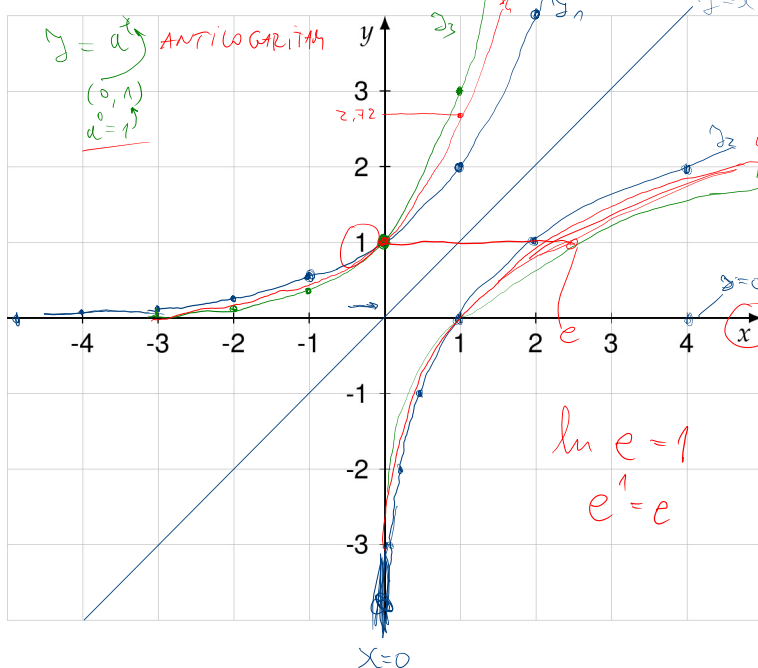
Skicirati grafike funkcija $y_1 = \sin x$, $y_2 = \cos x$, $y_3 = \sin(2x)$, $y_4 = \arcsin x$. ←



Skicirati grafike funkcija $y_1 = \tan x$, $y_2 = \arctan x$. Kotirati bitne tačke.



Skicirati grafike funkcija $y_1 = 2^x$, $y_2 = \log_2 x$, $y_3 = 3^x$, $y_4 = e^x$.



x	-3	-2	-1	0	1	2	$\log_2 x$
2^x	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	x
3^x	$\frac{1}{27}$	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9	$\log_3 x$

$$2^x: \mathbb{R} \rightarrow (0, +\infty)$$

$$\log_2 x: (0, +\infty) \rightarrow \mathbb{R}$$

$$\lim_{x \rightarrow -\infty} 2^x = 2^{-\infty} = \frac{1}{2^{\infty}} = \frac{1}{\infty} = 0$$

$$y=0 \text{ is a horizontal asymptote for } y=2^x.$$

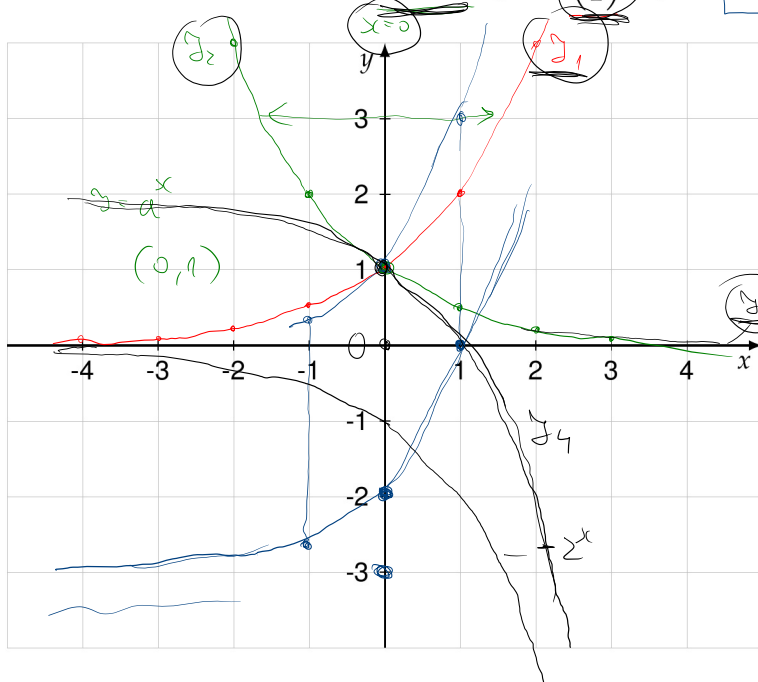
$$\lim_{x \rightarrow 0^+} \log_2 x = -\infty$$

? $e^{2.72}$ Euler $x = \ln y = \log_2 y$

$$\ln x: (0, +\infty) \rightarrow \mathbb{R}$$

$$\lim_{x \rightarrow 0^+} \ln x = -\infty$$

Skicirati grafike funkcija $y_1 = 2^x$, $y_2 = \left(\frac{1}{2}\right)^x$, $y_3 = 3^x - 3$, $y_4 = 2 - 2^x$.



x	-3	-2	-1	0	1	2	3
$\left(\frac{1}{2}\right)^x$	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$

3^x	$\frac{1}{27}$	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9	27
$3^x - 3$	$-\frac{26}{27}$	$-\frac{8}{9}$	$-\frac{2}{3}$	0	0	6	24

Važne granične vrednosti

$$(2)^n \rightarrow \infty$$

$$\frac{2}{2} \cdot \frac{1}{\frac{1}{h}} = \frac{1}{h}$$

Granična vrednost funkcije

$$\lim_{x \rightarrow a} f(x) = l \Leftrightarrow$$

$$\forall \epsilon > 0 \exists \delta > 0 (0 < |x - a| < \delta \Rightarrow |f(x) - l| < \epsilon)$$

Ako granične vrednosti postoje, važi:

$$\lim_{x \rightarrow a} (\alpha f(x) + \beta g(x)) = \alpha \lim_{x \rightarrow a} f(x) + \beta \lim_{x \rightarrow a} g(x)$$

$$\lim_{x \rightarrow a} (f(x) \cdot g(x)) = \lim_{x \rightarrow a} f(x) \cdot \lim_{x \rightarrow a} g(x)$$

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)}, \quad \lim_{x \rightarrow a} g(x) \neq 0$$

$$\lim_{x \rightarrow a} h(f(x)) = h\left(\lim_{x \rightarrow a} f(x)\right), \quad h \text{ neprekidno.}$$

Važne granične vrednosti

$$1. \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$2. \lim_{x \rightarrow \pm\infty} \left(1 + \frac{1}{x}\right)^x = e$$

$$3. \lim_{x \rightarrow 0} (1 + x)^{\frac{1}{x}} = e$$

$$4. \lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1$$

$$5. \lim_{x \rightarrow 0} \frac{\ln(1+x)}{x} = 1$$

$$6. \lim_{x \rightarrow \infty} \arctan x = \pi/2$$

Izračunati:

$$1. \lim_{x \rightarrow \infty} \frac{x^2 + 3x}{2x^2 - 4x} = \lim_{x \rightarrow \infty} \frac{\frac{x^2}{x^2} + \frac{3x}{x^2}}{\frac{2x^2}{x^2} - \frac{4x}{x^2}} = \lim_{x \rightarrow \infty} \frac{1 + \frac{3}{x}}{2 - \frac{4}{x}} = \frac{1}{2}$$

$$2. \lim_{x \rightarrow \infty} \frac{x^2 + 3x}{2x^2 - 4x} = \lim_{x \rightarrow \infty} \frac{1 + \frac{3}{x}}{2 - \frac{4}{x}} = \frac{1}{2}$$

$$3. \lim_{x \rightarrow \infty} \frac{x^2 + 3x}{2x^2 - 4x} = +\infty$$

$$4. \lim_{x \rightarrow 0} \frac{x^2 + 3x}{2x^2 - 4x} = \lim_{x \rightarrow 0} \frac{x + 3}{2x - 4} = -\frac{3}{4}$$

$$5. \lim_{x \rightarrow 0} \frac{x^2 + 3x}{2x^2 - 4x} = -\frac{3}{4}$$

$$6. \lim_{x \rightarrow 0} \frac{x^3 + 3}{2x^2 - 4x} = -\infty$$

$$x = 0.1 \quad 2 \times 0.01 = 4 \times 0.1 = 0.38$$

Granična vrednost funkcije

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$$5. \lim_{x \rightarrow 0} \frac{\ln(1+x)}{x} = 1$$

$$6. \lim_{x \rightarrow \infty} \arctan x = \pi/2$$

Izračunati:

$$1. \lim_{x \rightarrow 2} \frac{x^2 - 4}{x^2 - 5x + 6} = \lim_{x \rightarrow 2} \frac{(x-2)(x+2)}{(x-3)(x-2)} = \lim_{x \rightarrow 2} \frac{x+2}{x-3} = \frac{4}{-1} = -4$$

$$2. \lim_{x \rightarrow 4^+} \frac{\sqrt{x-2}}{\sqrt{x-4}} = \lim_{x \rightarrow 4^+} \frac{\sqrt{x-2} \cdot \sqrt{x-4}}{\sqrt{x-4} \cdot \sqrt{x-4}} = \lim_{x \rightarrow 4^+} \frac{(x-2) \cdot \sqrt{x-4}}{(\sqrt{x-4})^2} = \frac{0}{0} = 0$$

$$3. \lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1}{x} = \lim_{x \rightarrow 0} \frac{\sqrt{1+x} + 1}{\sqrt{1+x} + 1} = \lim_{x \rightarrow 0} \frac{1}{\sqrt{1+x} + 1} = \frac{1}{2}$$

$$4. \lim_{x \rightarrow 0} \frac{\tan x}{x} = \lim_{x \rightarrow 0} \frac{\sin x}{\cos x} = \lim_{x \rightarrow 0} \frac{\sin x}{x} \cdot \lim_{x \rightarrow 0} \frac{1}{\cos x} = 1 \cdot 1 = 1$$

$$5. \lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = \lim_{x \rightarrow 0} \frac{2 \sin^2 \frac{x}{2}}{(\frac{x}{2})^2 \cdot 4} = \frac{2}{4} \cdot \lim_{x \rightarrow 0} \left(\frac{\sin \frac{x}{2}}{\frac{x}{2}} \right)^2 = \frac{1}{2} \cdot \left(\lim_{t \rightarrow 0} \frac{\sin t}{t} \right)^2 = \frac{1}{2} \cdot 1^2 = \frac{1}{2}$$



$$8. \lim_{x \rightarrow 0} \frac{\arcsin x}{x} = 1$$

$$9. \lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = \frac{1}{2}$$

1. Koristeći kalkulator rešiti jednačinu $1.16x^2 - 0.32x - 1.361364 = 0$.

$$a = 1.16 \quad b = -0.32 \quad c = -1.361364$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x_1 = (+0.32 - 2.5336) / 2 / 1.16 = -0.954138$$

$$x_2 = (0.32 + 2.5336) / 2 / 1.16 = 1.230000$$

2. Koristeći kalkulator izračunati graničnu vrednost $\lim_{n \rightarrow \infty} (\sqrt{n^2 + 4n + 1} - \sqrt{n^2 + n}) = 1.5 = \frac{3}{2}$

$$(a-b) \cdot \frac{a+b}{a+b} = \frac{a^2 - b^2}{a+b}$$

$$a_n = \sqrt{10401} - \sqrt{10100} = 1.4865 \approx 1.5$$

$$a_{1000} = \sqrt{1004001} - \sqrt{1001000} = 1.4986$$

Dođe li $a_{1000} =$

3. Koristeći kalkulator izračunati graničnu vrednost $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = \frac{1}{2}$

$$f(x) = \frac{1 - \cos x}{x^2} = \frac{1 - \cos 0.01}{0.0001} = 0.49996 \approx \frac{1}{2}$$

$x = 0.01$

Izvod funkcije

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

Ako funkcija ima prvi izvod u tački x , kažemo da je **diferencijabilna** u toj tački.

Osobine izvoda

1. $(\alpha f(x) + \beta g(x))' = \alpha f'(x) + \beta g'(x)$
2. $(f(x) \cdot g(x))' = f'(x)g(x) + f(x)g'(x)$
3. $\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)}$
4. $(f(g(x)))' = f'(g(x))g'(x)$

Pokazati po definiciji da je $(x^2 - x + 1)' = 2x - 1$.

$$\begin{aligned} \lim_{\Delta x \rightarrow 0} \frac{(x + \Delta x)^2 - (x + \Delta x) + 1 - (x^2 - x + 1)}{\Delta x} &= \\ = \lim_{\Delta x \rightarrow 0} \frac{x^2 + 2x\Delta x + \Delta x^2 - x - \Delta x + 1 - x^2 + x - 1}{\Delta x} &= \\ = \lim_{\Delta x \rightarrow 0} \left(\frac{\Delta x(2x - 1)}{\Delta x} + \Delta x \right) &= 2x - 1 \end{aligned}$$

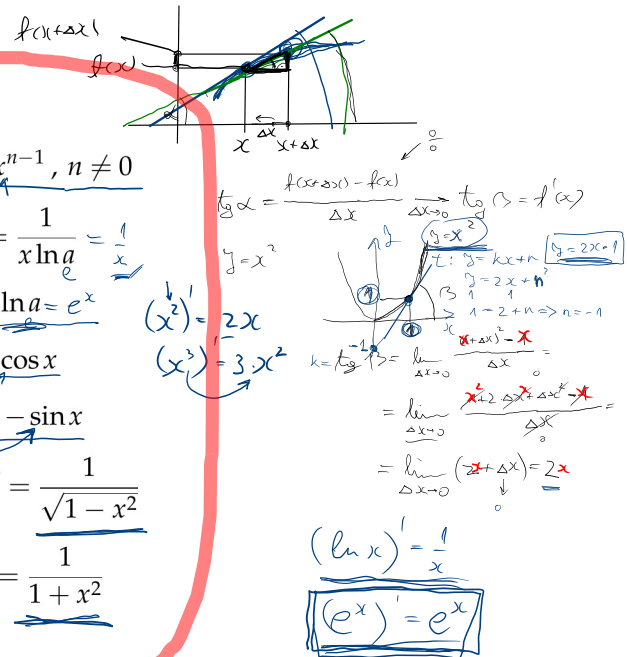
Naći po definiciji izvod funkcije $f(x) = \sqrt{x}$

$$\begin{aligned} (\sqrt{x})' &= \lim_{\Delta x \rightarrow 0} \frac{\sqrt{x + \Delta x} - \sqrt{x}}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{\sqrt{x + \Delta x} + \sqrt{x}}{\sqrt{x + \Delta x} + \sqrt{x}} \cdot \lim_{\Delta x \rightarrow 0} \frac{x + \Delta x - x}{\Delta x (\sqrt{x + \Delta x} + \sqrt{x})} = \\ &= \lim_{\Delta x \rightarrow 0} \frac{1}{\sqrt{x + \Delta x} + \sqrt{x}} \cdot \lim_{\Delta x \rightarrow 0} \frac{\Delta x}{\Delta x (\sqrt{x + \Delta x} + \sqrt{x})} = \frac{1}{2\sqrt{x}} \end{aligned}$$

$(\sqrt{x})' = (x^{\frac{1}{2}})' = \frac{1}{2} \cdot x^{\frac{1}{2}-1} = \frac{1}{2} \cdot \frac{1}{x^{\frac{1}{2}}} = \frac{1}{2\sqrt{x}}$

Tablica izvoda

- i) $c' = 0$
- ii) $(x^n)' = nx^{n-1}, n \neq 0$
- iii) $(\log_a x)' = \frac{1}{x \ln a}$
- iv) $(a^x)' = a^x \ln a = e^x$
- v) $(\sin x)' = \cos x$
- vi) $(\cos x)' = -\sin x$
- vii) $(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$
- viii) $(\arctg x)' = \frac{1}{1+x^2}$



Izvod funkcije

Tablica izvoda

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$$3. \left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)}$$

$$4. (f(g(x)))' = f'(g(x)) \cdot g'(x)$$

$$1. y = x^{\frac{1}{2}} \sqrt{x} - 3 \sin x - \frac{1}{x} + \ln x^2, \quad y' = \left(x^{\frac{3}{2}}\right)' - 3 \cdot \cos x + \frac{1}{x^2} + 2 \cdot (\ln x)'$$

$$y' = \frac{3}{2} \cdot \sqrt{x} - 3 \cdot \cos x + \frac{1}{x^2} + \frac{2}{x}$$

$$2. y = (x^2 + x)^2 - \frac{3}{e^{3+x}} + \sqrt{2x}, \quad y' = (x^4 + 2x^3 + x^2)' - (e^3 \cdot e^x)' + (\sqrt{2} \cdot \sqrt{x})'$$

$$y' = 4x^3 + 6x^2 + 2x - e^3 \cdot e^x + \frac{1}{\sqrt{2x}}$$

$$3. y = \frac{x}{\sqrt{x}} + 2 \cos x + \ln(2x), \quad y' =$$

$$\ln 2 + \ln x$$

$$y' = 2 \cdot (x^2 + x) \cdot (2x + 1) - e^{3+x} \cdot 1 + \frac{1}{2 \cdot \sqrt{2x}} \cdot 2$$

$$4. y = \frac{1}{x\sqrt{x}} - \frac{1}{e^{-x}} + \log_2 x, \quad y' =$$

$$i) c' = 0$$

$$ii) (x^n)' = nx^{n-1}, \quad n \neq 0$$

$$iii) (\log_a x)' = \frac{1}{x \ln a}$$

$$iv) (a^x)' = a^x \ln a$$

$$v) (\sin x)' = \cos x$$

$$vi) (\cos x)' = -\sin x$$

$$vii) (\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$$

$$viii) (\operatorname{arctg} x)' = \frac{1}{1+x^2}$$

$$\left(\frac{1}{x}\right)' = -\frac{1}{x^2}$$

$$(x^{-1})' = -x^{-2}$$

$$(e^x)' = e^x$$

$$(\sqrt{x})' = \frac{1}{2\sqrt{x}}$$

$$(x^2)' = 2x$$

$$\frac{\sqrt{2}}{2\sqrt{x}}$$

$$x' = 1$$