Granična vrednost niza

$$A = \lim_{n\to\infty} a_n$$
$$\Leftrightarrow$$
$$\forall \varepsilon > 0\,,\ \exists n_0 \in \mathbb{N}\,,\ \forall n \geq n_0\,,\ |A - a_n| < \varepsilon$$

Određeni oblici

$\infty \cdot \infty = \infty\,,\qquad \infty + \infty = \infty\,,\qquad \infty^{\infty} = \infty\,,$

$\frac{1}{\pm 0} = \pm\infty\,,\qquad \frac{1}{\pm\infty} = 0\,,\qquad \frac{0}{\pm\infty} = 0\,,$

$0^{\infty} = 0\,,\qquad \frac{\infty}{\pm 0} = \pm\infty\,.$

Neodređeni oblici

$\infty - \infty\,,\ 0 \cdot \infty\,,\ \frac{0}{0}\,,\ \frac{\infty}{\infty}\,,\ 1^{\infty}\,,\ 0^{0}\,,\ \infty^{0}\,.$

Važne granične vrednosti

1. $\displaystyle \lim_{n\to\infty} \frac{1}{n^{\alpha}} = \begin{cases} 0, & \alpha > 0 \\ 1, & \alpha = 0 \\ +\infty, & \alpha < 0 \end{cases}$

2. $\displaystyle \lim_{n\to\infty} q^{n} = \begin{cases} 0, & |q| < 1 \\ 1, & q = 1 \\ +\infty, & q > 1 \end{cases}$

3. $\displaystyle \lim_{\substack{n\to\infty \\ f(n)\to\pm\infty}} \left(1 + \frac{1}{f(n)}\right)^{g(n)} = e^{\lim_{n\to\infty} \frac{g(n)}{f(n)}}$

4. $\displaystyle \lim_{n\to\infty} \frac{n^{\alpha}}{a^{n}} = 0$, za $a > 1$

5. $\displaystyle \lim_{n\to\infty} (q^{n} - r^{n}) = +\infty$, za $q > r > 1$

6. $\displaystyle \lim_{n\to\infty} (n^{\alpha} - n^{\beta}) = +\infty$, za $\alpha > \beta > 0$

$$\frac{a}{b} = \frac{1}{\frac{b}{a}}$$

Izračunati:

4. $\displaystyle \lim_{n\to\infty} \frac{1.1^{n}}{100^{n}} = \lim_{n\to\infty} \left(\frac{1,1}{100}\right)^{n}$

$0,011$

$$= \lim_{n\to\infty} \frac{1,1^{n}}{n^{100}} = \lim_{n\to\infty} \frac{1}{\frac{n^{100}}{1,1^{n}}} = \frac{1}{\lim_{n\to\infty} \frac{n^{100}}{1,1^{n}}} = \frac{1}{0} = \infty$$

5. $\displaystyle \lim_{n\to\infty} \frac{\sqrt{n^2+n+1} - \sqrt{n^2+3n-1}}{n+3}$

$$(a-b)(a+b) = a^2 - b^2$$

$$\cdot \frac{\sqrt{n^2+n+1} + \sqrt{n^2+3n-1}}{\sqrt{n^2+n+1} + \sqrt{n^2+3n-1}} =$$

$$= \lim_{n\to\infty} \frac{n^2 + n + 1 - (n^2 + 3n - 1)}{(n+3)\cdot\left(\sqrt{n^2+n+1} + \sqrt{n^2+3n-1}\right)} =$$

$$= \lim_{n\to\infty} \frac{-2n+2}{(n+3)\cdot\left(\sqrt{} + \sqrt{}\right)} = \frac{\frac{1}{n}}{\frac{1}{n}}$$

$$= \lim_{n\to\infty} \frac{-2 + \frac{2}{n}}{\left(1 + \frac{3}{n}\right)\cdot\left(\sqrt{} + \sqrt{}\right)} =$$

$$= \frac{-2}{\infty} = 0$$

Granična vrednost funkcije

$$\lim_{x\to a} f(x) = l \Leftrightarrow$$

$$\forall \epsilon > 0\ \exists \delta > 0\ (0 < |x-a| < \delta \Rightarrow |f(x) - l| < \epsilon)$$

Ako granične vrednosti postoje, važi:

$$\lim_{x\to a}(\alpha f(x) + \beta g(x)) = \alpha \lim_{x\to a} f(x) + \beta \lim_{x\to a} g(x)$$

$$\lim_{x\to a}(f(x)\cdot g(x)) = \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x)$$

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}, \quad \lim_{x\to a} g(x) \neq 0$$

$-\infty$ $\quad$ $0\ 1\ 2$ $\quad$ $+\infty$

$$\lim_{x\to a} h(f(x)) = h\left(\lim_{x\to a} f(x)\right), \ h \text{ neprekidno.}$$

Važne granične vrednosti

1. $\lim_{x\to 0}\frac{\sin x}{x} = 1$
2. $\lim_{x\to\pm\infty}\left(1+\frac{1}{x}\right)^x = e$
3. $\lim_{x\to 0}(1+x)^{\frac{1}{x}} = e$
4. $\lim_{x\to 0}\frac{e^x-1}{x} = 1$
5. $\lim_{x\to 0}\frac{\ln(1+x)}{x} = 1$
6. $\lim_{x\to\infty}\arctan x = \pi/2$

Izračunati:

1. $\lim_{x\to\infty}\frac{x^2+3x}{2x^3-4x}$, $\cdot\frac{\frac{1}{x^3}}{\frac{1}{x^3}} = \lim_{x\to\infty}\frac{\frac{1}{x}+\frac{3}{x^2}}{2-\frac{4}{x^2}} = \frac{0}{2} = 0$

   $\lim_{x\to\infty}\frac{x^4+3x}{2x^3-4x}\cdot\frac{\frac{1}{x^3}}{\frac{1}{x^3}} = \lim_{x\to\infty}\frac{x+\frac{3}{x^2}}{2-\frac{4}{x^2}} = +\infty$

2. $\lim_{x\to\infty}\frac{x^2+3x}{2x^2-4x}$, $\cdot\frac{\frac{1}{x^2}}{\frac{1}{x^2}} = \lim_{x\to\infty}\frac{1+\frac{3}{x}}{2-\frac{4}{x}} = \frac{1+0}{2-0} = \frac{1}{2}$

   $\frac{1}{2}$

3. $\lim_{x\to\infty}\frac{x^3+3x}{2x^2-4x}$, $\frac{\frac{1}{x^2}}{\frac{1}{x^2}} = \infty$

   $\frac{0+0}{0-0} = \frac{0}{0}$??

4. $\lim_{x\to 0}\frac{x^2+3x}{2x^3-4x}$, $\cdot\frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x\to 0}\frac{x+3}{2x^2-4} = \frac{3}{-4} = -\frac{3}{4}$

5. $\lim_{x\to 0}\frac{x^2+3x}{2x^2-4x}$,

6. $\lim_{x\to 0}\frac{x^3+3}{2x^2-4x}$.

Granična vrednost funkcije

$$\lim_{x\to a} f(x) = l \Leftrightarrow$$

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$$\lim_{x\to a}(f(x)\cdot g(x)) = \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x)$$

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}, \quad \lim_{x\to a} g(x) \neq 0$$

$$\lim_{x\to a} h(f(x)) = h\left(\lim_{x\to a} f(x)\right), \ h \text{ neprekidno.}$$

Važne granične vrednosti

1. $\lim_{x\to 0}\frac{\sin x}{x} = 1$
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4. $\lim_{x\to 0}\frac{e^x - 1}{x} = 1$
5. $\lim_{x\to 0}\frac{\ln(1+x)}{x} = 1$
6. $\lim_{x\to\infty}\arctan x = \pi/2$

$a \neq 0$

$ax^2 + bx + c = a\cdot(x-x_1)(x-x_2)$

$x_{1/2} = \frac{-b \pm d}{2a}$

$d = \sqrt{b^2 - 4ac}$

Izračunati:

1. $\lim_{x\to 2}\frac{x^2-4}{x^2-5x+6} = \lim_{x\to 2}\frac{(x-2)\cdot(x+2)}{(x-2)(x-3)} = \lim_{x\to 2}\frac{x+2}{x-3} = \frac{4}{-1} = -4$

$\frac{2^2-4}{4-10+6}$

$x_{1/2} = \frac{5 \pm \sqrt{25-24}}{2}$, $x_1 = 2$, $x_2 = 3$

$\frac{0}{\ } = 0$

$a^2 - b^2 = (a-b)(a+b)$

2. $\lim_{x\to 4^+}\frac{\sqrt{x}-2}{\sqrt{x-4}} \cdot \frac{\sqrt{x}+2}{\sqrt{x}+2}\cdot\frac{\sqrt{x-4}}{\sqrt{x-4}} = \lim_{x\to 4^+}\frac{(x-4)\sqrt{x-4}}{(x-4)(\sqrt{x}+2)} = \lim_{x\to 4^+}\frac{\sqrt{x-4}}{\sqrt{x}+2} = \frac{0}{4} = 0$

$\frac{0}{0}$

$(\sqrt{x})'\big|_{x=1} = \frac{1}{2\sqrt{x}}\big|_{x=1}$

3. $\lim_{x\to 0}\frac{\sqrt{1+x}-1}{x}$. $\frac{1-1}{0} = \frac{0}{0} = \lim_{x\to 0}\frac{\sqrt{1+x}-1}{x}\cdot\frac{\sqrt{1+x}+1}{\sqrt{1+x}+1} =$

$= \lim_{x\to 0}\frac{1+x-1}{x(\sqrt{1+x}+1)} = \lim_{x\to 0}\frac{1}{\sqrt{1+x}+1} = \frac{1}{1+1} = \frac{1}{2}$

$1 - 1 = 0$ $\quad \frac{x}{x} = 1$

4. $\lim_{x\to 0}\frac{\tan x}{x} = \lim_{x\to 0}\frac{\frac{\sin x}{\cos x}}{x} = \lim_{x\to 0}\frac{\sin x}{x}\cdot\lim_{x\to 0}\frac{1}{\cos x}$

$= \lim_{x\to 0}\frac{1}{\cos x} = \frac{1}{1} = 1$

$1 - \cos x =$

$1 = \cos^2 x + \sin^2 x$

$\cos(2x) = \cos^2 x - \sin^2 x$

$1 - \cos(2x) = 2\cdot\sin^2 x$

$\boxed{1 - \cos x = 2\cdot\sin^2\frac{x}{2}}$

5. $\lim_{x\to 0}\frac{1-\cos x}{x^2} = \lim_{x\to 0}\frac{2\cdot\sin^2\frac{x}{2}}{4\cdot\left(\frac{x}{2}\right)^2} =$

$\frac{0}{0}$

$\frac{x}{2} = t \quad x\to 0 \quad t\to 0$

$= \frac{1}{2}\cdot\lim_{x\to 0}\left(\frac{\sin\frac{x}{2}}{\frac{x}{2}}\right)^2 = \frac{1}{2}\left(\lim_{t\to 0}\frac{\sin t}{t}\right)^2 = \frac{1}{2}$

1. Koristeći kalkulator rešiti jednačinu $1.16\,x^2 - 0.32\,x - 1.361364 = 0$.

$a = 1.16,\ b = 0.32,\ c = 1.361364,\ d = b^2 - 4ac$

$0.32\ \boxed{x^2} + 4\ \boxed{\times}\ 1.16\ \boxed{\times}\ 1.361364\ \boxed{=}\ 6.419129$ 896

$x_1 = \dfrac{-b - \sqrt{d}}{2a} = -0.32 - \sqrt{6.41\ldots}\ \boxed{=}\ \boxed{\div}\ 2\ \boxed{\times}\ 1.16\ \boxed{=}$

2. Koristeći kalkulator izračunati graničnu vrednost $\lim_{n\to\infty}\left(\sqrt{n^2+4n+1} - \sqrt{n^2+n}\right)$. $\boxed{= 1.23}$

a_n $\boxed{0.9838093845}$

$A = \lim_{n\to\infty} a_n \quad \forall \varepsilon\ \exists n_0\ \forall n\ n \geq n_0 \Rightarrow |A - a_n| < \varepsilon$

$n_0(\varepsilon)$

$n = 1000 \quad a_{1000} = \sqrt{1004001} - \sqrt{1001000}$

$a_{10000} = \sqrt{100040001} - \sqrt{100010000} = 1{,}498628$ $= 1{,}49860$ $\boxed{A = \frac{3}{2}}$

3. Koristeći kalkulator izračunati graničnu vrednost $\lim_{x\to 0}\dfrac{1-\cos x}{x^2}$.

$x = 0{,}01$

$f(0{,}01) = \dfrac{1 - \cos 0{,}01}{0{,}0001} = \dfrac{1 - 0{,}99995000}{0{,}0001}$ $f(x)$

$= \dfrac{0{,}0000499996}{0{,}0001} = 0{,}499996 \approx 0{,}5 = \frac{1}{2}$

$$\lim_{n\to\infty}\left(\sqrt{n^2+4n+1}-\sqrt{n^2+n}\right)\cdot\frac{\sqrt{}+\sqrt{}}{\sqrt{}+\sqrt{}}=$$

$$\boxed{\infty-\infty}$$

$$=\lim_{n\to\infty}\frac{\cancel{n^2}+4n+1-\cancel{n^2}-n}{\sqrt{n^2+4n+1}+\sqrt{n^2+n}}=$$

$$=\lim_{n\to\infty}\frac{3n+1}{\sqrt{n^2+4n+1}+\sqrt{n^2+n}}\quad\boxed{\frac{\infty}{\infty}}\quad\cdot\frac{\frac{1}{n}}{\frac{1}{\sqrt{n^2}}}$$

$$=\lim_{n\to\infty}\frac{3+\frac{1}{n}}{\sqrt{1+\frac{4}{n}+\frac{1}{n^2}}+\sqrt{1+\frac{1}{n}}}=\frac{3}{2}$$

$$\frac{1}{n}\to 0$$

$$\lim_{x\to 0}\frac{1-\cos x}{x^2}=\lim_{x\to 0}$$

$$=\frac{1}{2}$$

Izvod funkcije

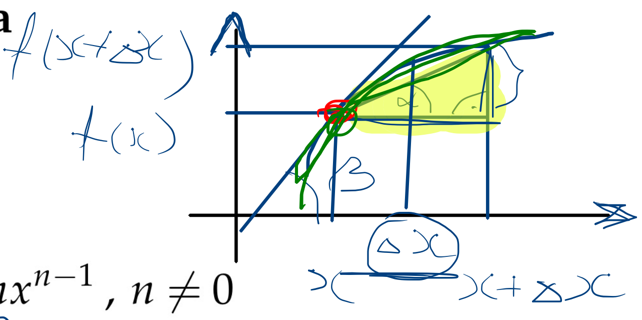
$$f'(x) = \lim_{\Delta x\to 0}\frac{\overbrace{f(x+\Delta x)-f(x)}^{\Delta f}}{\Delta x} \qquad \frac{0}{0}$$

Ako funkcija ima prvi izvod u tački x, kažemo da je **diferencijabilna** u toj tački.

Osobine izvoda

1. $(\alpha f(x) + \beta g(x))' = \alpha f'(x) + \beta g'(x)$
2. $(f(x)\cdot g(x))' = f'(x)g(x) + f(x)g'(x)$
3. $\left(\dfrac{f(x)}{g(x)}\right)' = \dfrac{f'(x)g(x) - f(x)g'(x)}{g^2(x)}$
4. $(f(g(x)))' = f'(g(x))g'(x)$

Tablica izvoda



i) $c' = 0$

ii) $(x^n)' = nx^{n-1}$, $n \neq 0$

iii) $(\log_a x)' = \dfrac{1}{x\ln a}$

iv) $(a^x)' = a^x \ln a$

v) $(\sin x)' = \cos x$

vi) $(\cos x)' = -\sin x$

vii) $(\arcsin x)' = \dfrac{1}{\sqrt{1-x^2}}$

viii) $(\operatorname{arctg} x)' = \dfrac{1}{1+x^2}$

Pokazati po definiciji da je $(\underbrace{x^2 - x + 1}_{f(x)})' = 2x - 1$.

$$f'(x) = \lim_{\Delta x\to 0}\frac{(x+\Delta x)^2 - (x+\Delta x) + 1 - (x^2 - x + 1)}{\Delta x}$$

$$= \lim_{\Delta x\to 0}\frac{\cancel{x^2} + 2x\Delta x + \Delta x^2 - \cancel{x} - \Delta x + \cancel{1} - \cancel{x^2} + \cancel{x} - \cancel{1}}{\Delta x} =$$

$$= \lim_{\Delta x\to 0}\left(\frac{\cancel{\Delta x}(2x-1)}{\cancel{\Delta x}} + \cancel{\Delta x}\right) = 2x - 1$$

$$\operatorname{tg}\alpha = \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$\left(x^{\frac{1}{2}}\right)' = \frac{1}{2}\cdot x^{\frac{1}{2}-1} = \frac{1}{2}\cdot x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$$

Naći po definiciji izvod funkcije $f(x) = \sqrt{x}$

$$f'(x) = \lim_{\Delta x\to 0}\frac{\sqrt{x+\Delta x} - \sqrt{x}}{\Delta x}\cdot\frac{\sqrt{x+\Delta x}+\sqrt{x}}{\sqrt{x+\Delta x}+\sqrt{x}} =$$

$$= \lim_{\Delta x\to 0}\frac{\cancel{x} + \cancel{\Delta x} - \cancel{x}}{\cancel{\Delta x}\cdot(\sqrt{x+\Delta x}+\sqrt{x})} = \lim_{\Delta x\to 0}\frac{1}{\sqrt{x+\Delta x}+\sqrt{x}} =$$

$$= \frac{1}{\sqrt{x}+\sqrt{x}} = \frac{1}{2\sqrt{x}}$$

Izvod funkcije

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

Ako funkcija ima prvi izvod u tački x, kažemo da je **diferencijabilna** u toj tački.

Osobine izvoda

$\ln e = \log_e e = 1$

1. $(\alpha f(x) + \beta g(x))' = \alpha f'(x) + \beta g'(x)$
2. $(f(x) \cdot g(x))' = f'(x)g(x) + f(x)g'(x)$
3. $\left(\dfrac{f(x)}{g(x)}\right)' = \dfrac{f'(x)g(x) - f(x)g'(x)}{g^2(x)}$
4. $(f(g(x)))' = f'(g(x))g'(x)$

Tablica izvoda

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$\boxed{(e^x)' = e^x \cdot \cancel{\ln e}}$

v) $(\sin x)' = \cos x$

vi) $\boxed{(\cos x)' = -\sin x}$

$(-\cos x)' = \sin x$

vii) $(\arcsin x)' = \dfrac{1}{\sqrt{1-x^2}}$

viii) $(\operatorname{arctg} x)' = \dfrac{1}{1+x^2}$

1. $y = x\sqrt{x} - 3\sin x - \dfrac{1}{x} + \ln x^2$, $\quad y' = \left(x^{\frac{3}{2}} - 3\cdot\sin x - x^{-1} + 2\ln x\right)' =$

$= \frac{3}{2} x^{\frac{3}{2}-1} - 3 \cdot \cos x - (-1) \cdot x^{-1-1} + 2 \cdot \frac{1}{x \cdot \ln e} =$

$= \frac{3}{2} \cdot \sqrt{x} - 3 \cdot \cos x + \frac{1}{x^2} + \frac{2}{x}$

2. $y = (x^2+x)^2 - e^{3+x} + \sqrt{2x}$, $\quad y' = \left(x^4 + 2 \cdot x^3 + x^2 - e^3 \cdot e^x + \sqrt{2} \cdot \sqrt{x}\right)' = 4 \cdot x^3 + 2 \cdot 3x^2 + 2 \cdot x - e^3 \cdot e^x + \sqrt{2} \cdot \frac{1}{2\sqrt{x}} = 4x^3 + 6x^2 + 2x - e^{x+3} + \frac{1}{\sqrt{2x}}$

3. $y = \dfrac{x}{\sqrt{x}} + 2\cos x + \ln(2x)$, $\quad y' =$

4. $y = \dfrac{1}{x\sqrt{x}} - \dfrac{1}{e^{-x}} + \log_2 x$, $\quad y' =$

Naći prvi izvod funkcije $y = f(x)$ i uprostiti dobijeni izraz

a) $y = x^2 \cos x + \sin x$

b) $y = \frac{e^x}{x^2} + 2x \ln x$

c) $y = \operatorname{tg} x$

d) $y = \ln \operatorname{tg}\left(\frac{x}{2} + \frac{\pi}{4}\right)$

e) $y = \frac{x^2+1}{x^2-x+1}$

f) $y = \frac{\ln(x-1)}{x^2-1}$

g) $y = \ln \frac{1+\operatorname{tg}\frac{x}{2}}{1-\operatorname{tg}\frac{x}{2}}$

$e^x \cdot x^{-2}$

$(x)' = x$

$(f \cdot g)' = f' \cdot g + f \cdot g'$

$\left(\frac{f}{g}\right)' = \frac{f' g - f g'}{g^2}$

a) $y' = (x^2)' \cdot \cos x + x^2 \cdot (\cos x)' + \cos x =$

$= 2x \cdot \cos x - x^2 \cdot \sin x + \cos x$

$= (2x+1) \cdot \cos x - x^2 \cdot \sin x$

b) $y' = \frac{e^x \cdot x^2 - e^x \cdot 2x}{x^4} + 2 \cdot \ln x + 2x \cdot \frac{1}{x}$

$= \frac{(x-2) \cdot e^x}{x^3} + 2 \cdot \ln x + 2$

c) $y' = \left(\frac{\sin x}{\cos x}\right)' = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$

Naći prvi izvod funkcije $y = f(x)$ i uprostiti dobijeni izraz

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e) $y = \dfrac{x^2+1}{x^2-x+1}$

f) $y = \dfrac{\ln(x-1)}{x^2-1}$

g) $y = \ln \dfrac{1+\operatorname{tg}\frac{x}{2}}{1-\operatorname{tg}\frac{x}{2}}$

x^2+1

x^2+2

?

a)
$$
\begin{aligned}
y' &= (x^2\cos x + \sin x)' \\
1. \Rightarrow \quad &= (x^2 \cos x)' + (\sin x)' \\
\text{v}),2. \Rightarrow \quad &= (x^2)'\cos x + x^2(\cos x)' + \cos x \\
\text{ii}),\ \text{vi}) \Rightarrow \quad &= 2x\cos x + x^2(-\sin x) + \cos x \\
&= (2x+1)\cos x - x^2 \sin x
\end{aligned}
$$

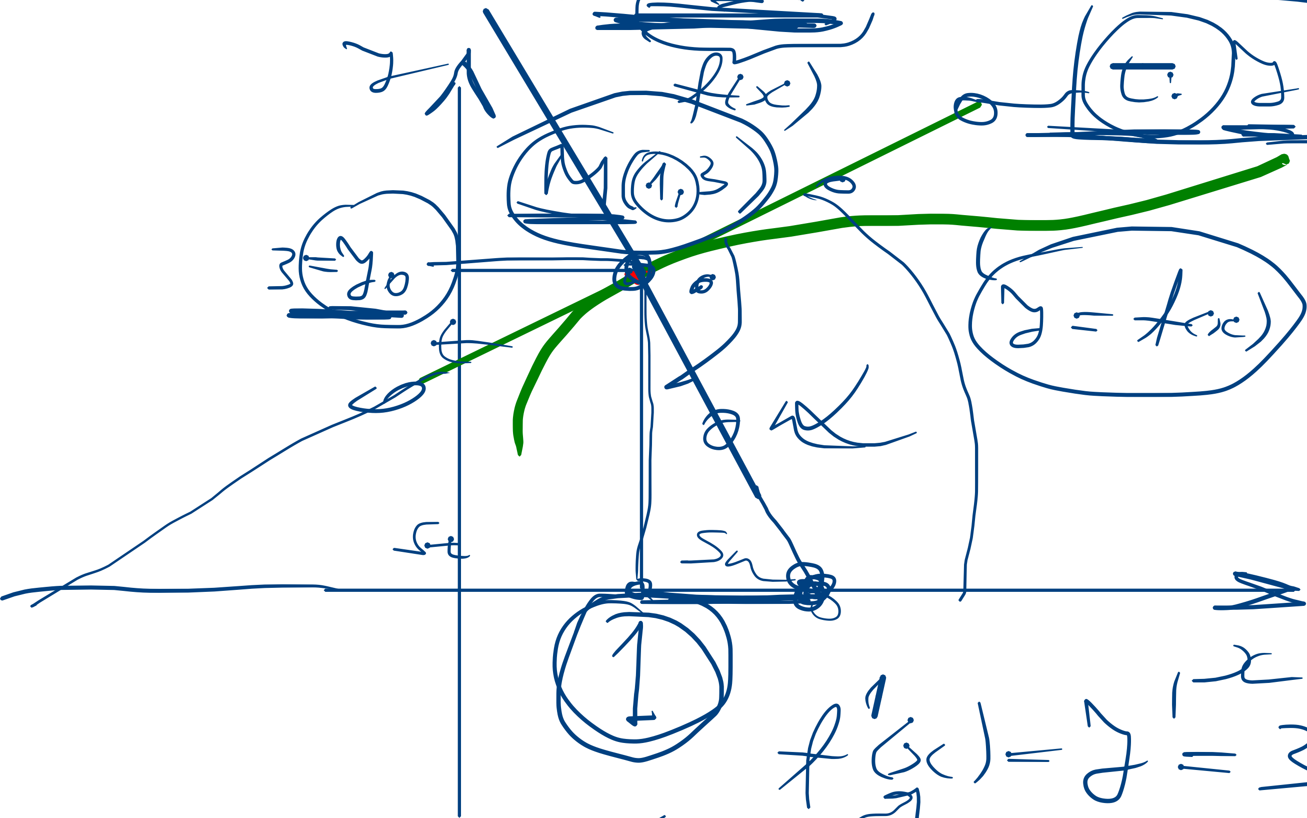
b)
$$
\begin{aligned}
y' &= (x^{-2}e^x + 2x\ln x)' \\
1. \Rightarrow \quad &= (x^{-2}e^x)' + (2x\ln x)' \\
2. \Rightarrow \quad &= (x^{-2})'e^x + x^{-2}(e^x)' + (2x)'\ln x + 2x(\ln x)' \\
\text{ii}),\ \text{iii}),\ \text{iv}) \Rightarrow \quad &= -2x^{-3}e^x + x^{-2}e^x + 2\ln x + 2x \cdot \frac{1}{x} \\
&= \frac{x-2}{x^3}e^x + 2\ln x + 2
\end{aligned}
$$

c)
$$
\begin{aligned}
y' &= \left(\frac{\sin x}{\cos x}\right)' \\
3. \Rightarrow \quad &= \frac{(\sin x)'\cos x - \sin x(\cos x)'}{\cos^2 x} \\
\text{v}),\ \text{vi}) \Rightarrow \quad &= \frac{\cos^2 x - (-\sin^2 x)}{\cos^2 x} = \frac{1}{\cos^2 x}
\end{aligned}
$$

d)
$$
\begin{aligned}
y' &= \Big(\underbrace{\ln}_{f} \underbrace{\operatorname{tg}\left(\tfrac{x}{2} + \tfrac{\pi}{4}\right)}_{g}\Big)' \\
\text{iii}),\ 4. \Rightarrow \quad &= \frac{1}{\operatorname{tg}\left(\frac{x}{2} + \frac{\pi}{4}\right)}\left(\operatorname{tg}\left(\frac{x}{2} + \frac{\pi}{4}\right)\right)' \\
\text{c}),\ 4. \Rightarrow \quad &= \frac{\cancel{\cos\left(\frac{x}{2} + \frac{\pi}{4}\right)}}{\sin\left(\frac{x}{2} + \frac{\pi}{4}\right)} \cdot \frac{1}{\cos^{\cancel{2}}\left(\frac{x}{2} + \frac{\pi}{4}\right)} \cdot \left(\frac{x}{2} + \frac{\pi}{4}\right)' \\
\text{i}),\ \text{ii}),\ \text{iii}) \Rightarrow \quad &= \frac{1}{\sin\left(\frac{x}{2} + \frac{\pi}{4}\right)\cos\left(\frac{x}{2} + \frac{\pi}{4}\right)} \cdot \frac{1}{2} \\
&= \frac{1}{\sin\left(x + \frac{\pi}{2}\right)} \\
&= \frac{1}{\cos x}
\end{aligned}
$$

e) $-\dfrac{x^2-1}{(x^2-x+1)^2}$ f) $\dfrac{x+1-2x\ln(x-1)}{(x^2-1)^2}$ g) $\dfrac{1}{\cos x}$

Za krivu $y = x^3 + x + 1$, u tački $M(1, y_0)$ krive, napisati jednačinu tangente i normale.



$$\boxed{t: y = kx + n}$$

$$k = \operatorname{tg} \alpha = f'(1)$$

$$y = f(x)$$

$$f'(x) = y' = 3x^2 + 1$$

$$k = f'(1) = 3 \cdot 1^2 + 1 = 4$$

$$y_0 = f(1) = 3 \qquad 3 = 4 \cdot 1 + n \Rightarrow n = -1$$

$$t: \boxed{y = 4x - 1} \qquad n: y = k_1 \cdot x + n_1$$

$$3 = -\frac{1}{4} \cdot 1 + n_1 \Rightarrow n_1 = \frac{13}{4} \qquad \boxed{k_1 = -\frac{1}{k}} = -\frac{1}{4}$$

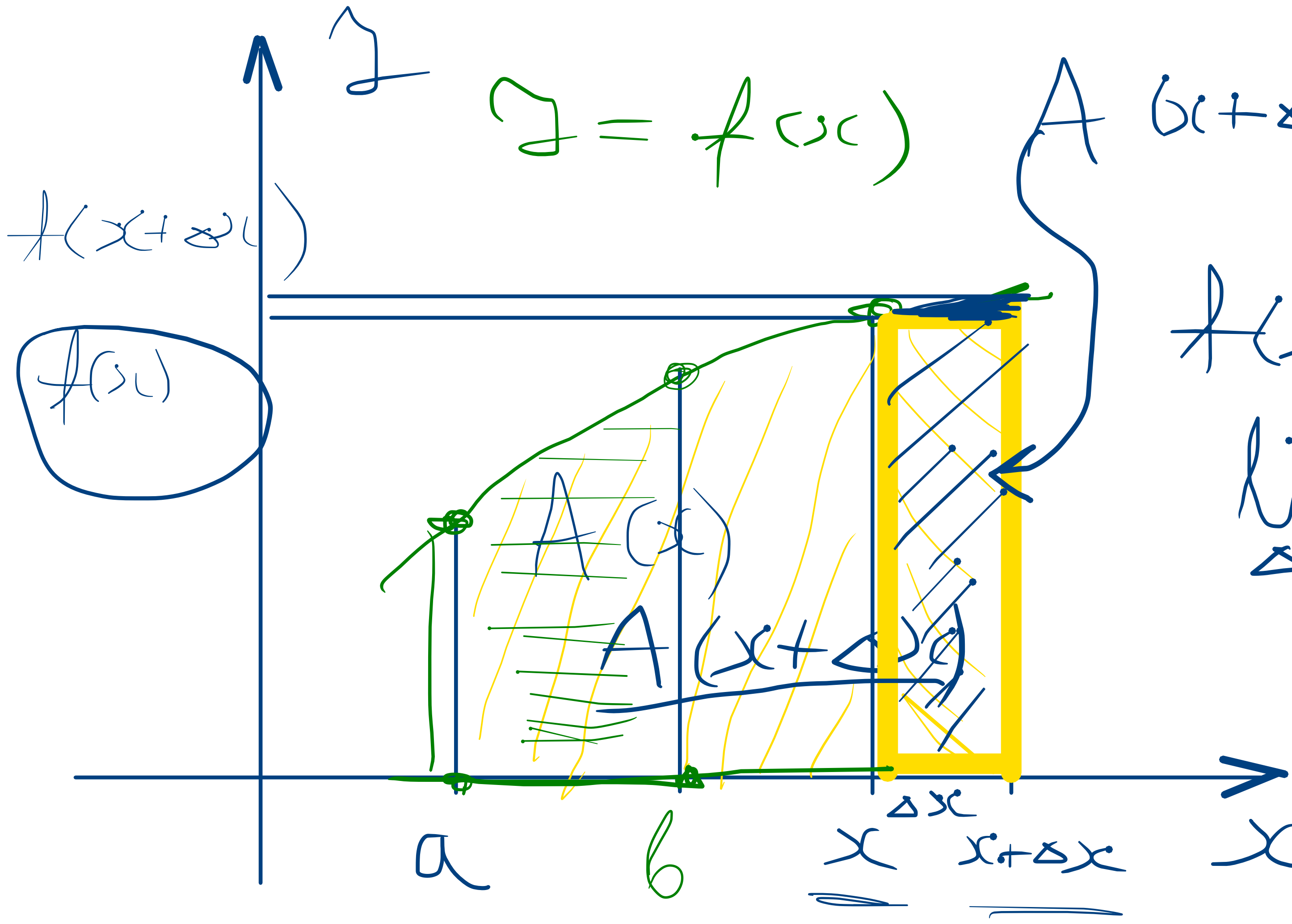
$$\boxed{n: y = -\frac{1}{4}x + \frac{13}{4}}$$

Za krivu $y = x^3 + x + 1$, u tački $M(1, y_0)$ krive, napisati jednačinu tangente i normale.
Nađimo prvi izvod posmatrane funkcije $y' = (x^3 + x + 1)' = 3x^2 + 1$. Za $x_0 = 1$ je $y_0 = y(x_0) = 1^3 + 1 + 1 = 3$ i $y'(x_0) = 3 \cdot 1^2 + 1 = 4$.
Jednačina tangente t glasi $y - y_0 = y'(x_0)(x - x_0)$, odnosno

$$t : y - 3 = 4(x - 1) \;\Leftrightarrow\; y = 4x - 4 + 3 = 4x - 1\,.$$

Jednačina normale n glasi $y - y_0 = -\frac{1}{y'(x_0)}(x - x_0)$, odnosno

$$n : y - 3 = -\frac{1}{4}(x - 1) \;\Leftrightarrow\; y = -\frac{1}{4}x + \frac{1}{4} + 3 = -\frac{1}{4}x + \frac{13}{4}\,.$$



$$A(x+\Delta x) - A(x) \approx f(x)\cdot\Delta x$$

$$f(x) \approx \frac{A(x+\Delta x) - A(x)}{\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{A(x+\Delta x) - A(x)}{\Delta x} =$$

$$f(x) = A'(x)$$

$$A(x) = F(x)$$

$$F'(x) = f(x)$$

F = PRIMITIVNA F.

$$(x^2)' = 2x$$

$$\int f(x)\,dx = F(x) + C$$

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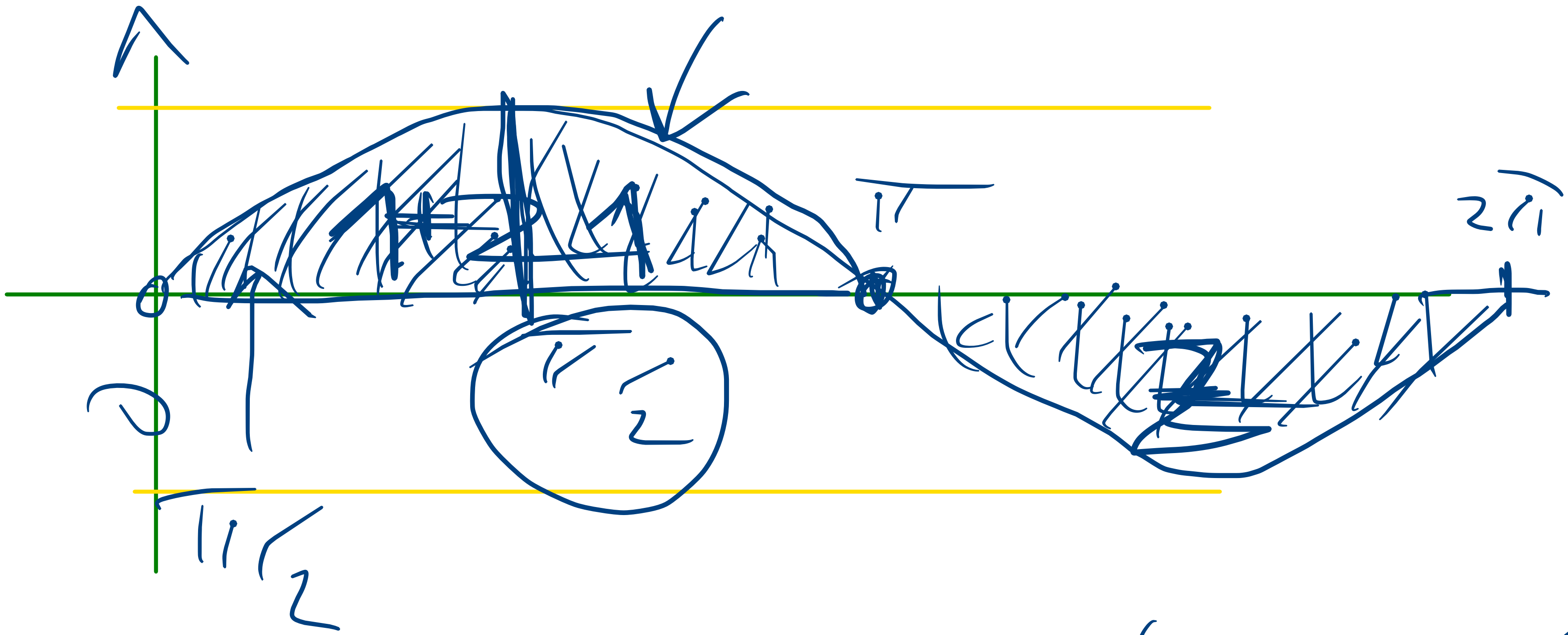
$$\int_a^b f(x)\,dx = F(b) - F(a)$$

a POVRŠINA IZMEĐU X-OSA KRIVE f i APSCISA $x=a$ i $x=b$



$$P = 2 \cdot \underbrace{\int_{0}^{\frac{\pi}{2}} \sin x \, dx}_{} = 2 \cdot \left(\cancel{-\cos\left(\frac{\pi}{2}\right)} + \cos 0 \right)$$

$$= 2 \cdot 1 = 2$$

$$\int \sin x \, dx = -\cos x + C$$

$$F(x) = -\cos x$$