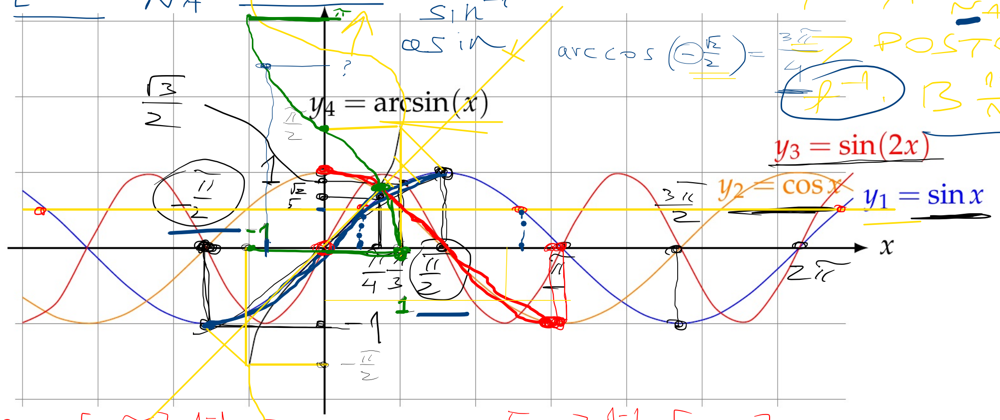
Skicirati grafike funkcija $y_1 = \sin x$, $y_2 = \cos x$, $y_3 = \sin(2x)$, $y_4 = \arcsin x$.

$\sin : \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \xrightarrow[NA]{1-1} [-1, 1]$ $\arcsin : [-1,1] \xrightarrow[NA]{1-1} \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\sin^{-1}$

$f : A \xrightarrow[NA]{1-1} B$ POŠTO $\exists f^{-1}$ $f^{-1} : B \xrightarrow[NA]{1-1} A$

$\arccos\left(-\frac{\sqrt{2}}{2}\right) = \frac{3\pi}{4}$



$\cos : [0, \pi] \xrightarrow[NA]{1-1} [-1, 1]$ $\arccos : [-1, 1] \xrightarrow[NA]{1-1} [0, \pi]$

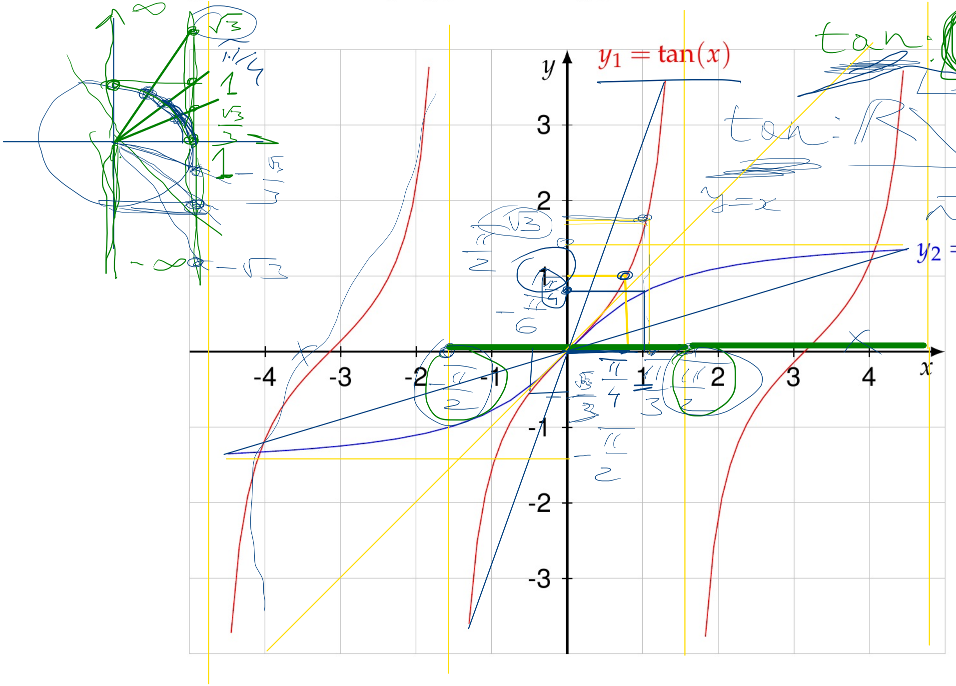
Kotirati na grafiku bitne tačke.

Skicirati grafike funkcija $y_1 = \tan x$, $y_2 = \arctan x$. Kotirati bitne tačke.

$\mathbb{R}$

$\tan : \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \xrightarrow[NA]{1-1} (-\infty, +\infty)$

$\tan : \mathbb{R} \setminus \left\{\frac{\pi}{2} + k\pi \mid k \in \mathbb{Z}\right\} \xrightarrow{NA} \mathbb{R}$



$y_2 = \arctan(x) : \mathbb{R} \to \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$f(-x) = -f(x)$

$f(x) = x^n$

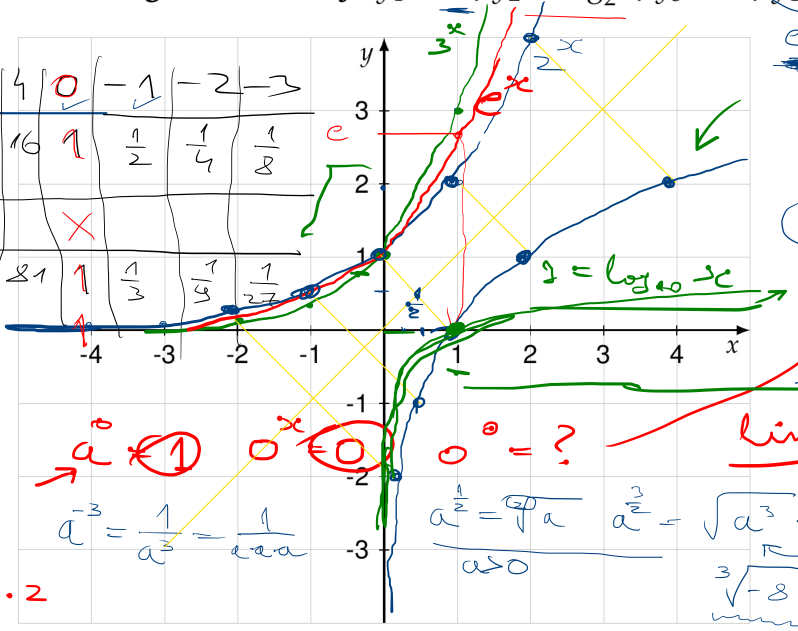
STEPENA FUNK

$f(-x) = f(x)$

PARNA F.

Skicirati grafike funkcija $y_1 = 2^x$, $y_2 = \log_2 x$, $y_3 = 3^x$, $y_4 = e^x$.

x	1	2	3	4	0	−1	−2	−3
y_1	2	4	8	16	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$
y_2					X			
y_3	3	9	27	81	1	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{27}$
y_4	e							



$e \approx 2{,}71828182845 90 \ldots$

Euler

$$\left(1+\frac{1}{n}\right)^n \to e$$

Limesi

$a^b = c$

$2^3 = 8 = 2 \cdot 2 \cdot 2$

$a^0 = \boxed{1}$ $\quad 0^x = \boxed{0}$ $\quad 0^0 = ?$

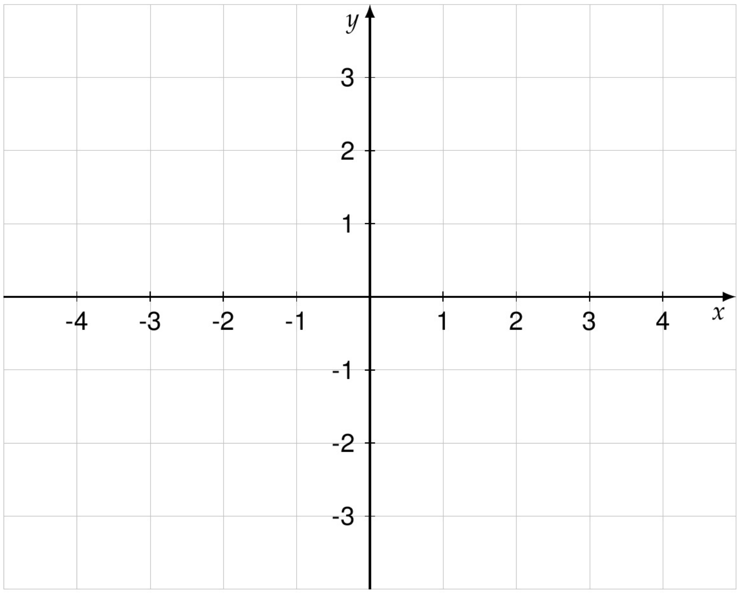
$a^{-3} = \frac{1}{a^3} = \frac{1}{a \cdot a \cdot a}$

$a^{\frac{1}{2}} = \sqrt[2]{a}$ $\quad a>0$

$a^{\frac{3}{2}} = \sqrt{a^3} = (\sqrt{a})^3$

$\sqrt[3]{-8} = -2$

Skicirati grafike funkcija $y_1 = 2^x$, $y_2 = \left(\frac{1}{2}\right)^x$, $y_3 = 3^x - 3$, $y_4 = 2 - 2^x$.



Stepenovanje

$a > 0 \qquad a^{\frac{m}{n}}$

$2^{1.5} = 2^{\frac{3}{2}} = \sqrt{2}^{\,3}$ ③

$= \underbrace{1.4142 \ldots}_{\in Ir \subset R}{}^{3}$

$= 2.8289$

$2^{-\frac{3}{2}} = \frac{1}{2^{3/2}} = \left(\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}\right)^{3} \approx 0,7070^{③} = 0,3536$

$= 0,354$

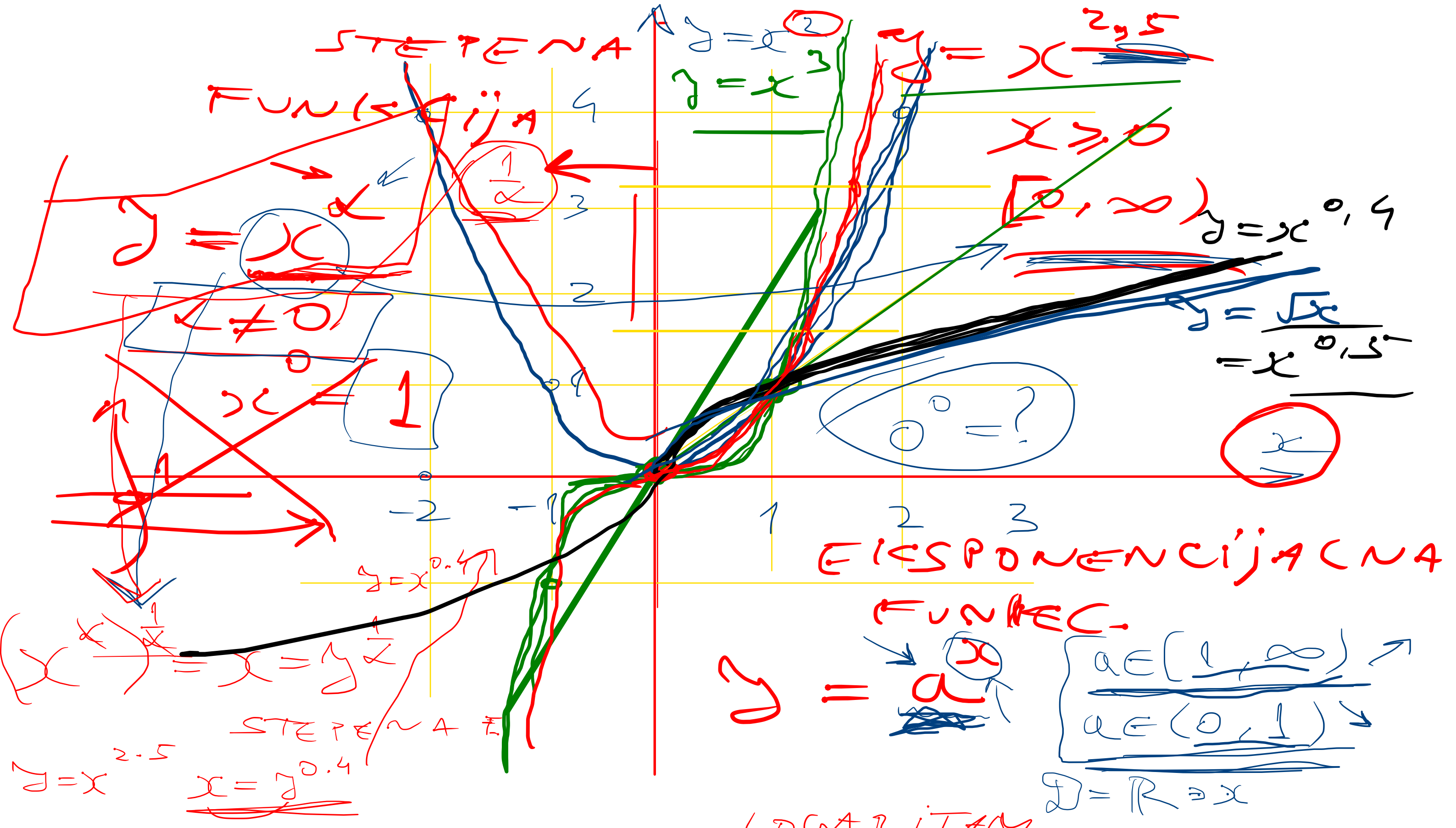
Ir $\quad m, n \in N$

$\Rightarrow$?

$2^{\sqrt{2}} = 2^{\cancel{\frac{m}{n}}}$

$2^{\frac{m_1}{n_1}}, 2^{\frac{m_2}{n_2}}, \ldots \longrightarrow \sqrt{2}$

$\ldots \longrightarrow 2^{\sqrt{2}}$



STEPENA FUNKCIJA

$y = x^{\alpha}$

$\frac{1}{\alpha}$

$\alpha \neq 0$

$x^{0} = 1$

$y = x^{2}$

$y = x^{3}$

$y = x^{2,5}$

$x \geq 0$

$[0, \infty)$

$y = x^{0,4}$

$y = \sqrt{x} = x^{0,5}$

$0^{0} = ?$

x

$(x^{\alpha})^{\frac{1}{\alpha}} = x = y^{\frac{1}{\alpha}}$

STEPENA F.

$y = x^{2.5}$ $x = y^{0.4}$

EKSPONENCIJALNA FUNKC.

$y = a^{x}$

$a \in (1, \infty)$

$a \in (0, 1)$

$D = \mathbb{R} \ni x$

LOGARITAM

OSNOVA

$$a^{b} = c \Leftrightarrow b = \log_{a} c$$

NUMERUS

ANTILOGARITAM

BRIGS 1700 TI

$a = 10$

2

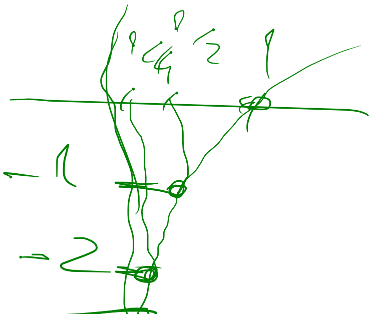
e

2,718

$2^3 = 8 \iff \log_2 8 = 3$

$\log_2 16 = 4$

$\log_2 1024 = 10$



$\log_2 \frac{1}{4} = -2$ $\qquad 2^{\boxed{-2}} = \frac{1}{4}$

$\log_2 \sqrt{2} = \boxed{\frac{1}{2}} \qquad 2^{\boxed{\frac{1}{2}}} = \sqrt{2}$

$\log_2 \sqrt{8} = \frac{3}{2} \qquad 2^{\boxed{\frac{3}{2}}} = \sqrt{8} = (8)^{\frac{1}{2}}$

$= (2^3)^{\frac{1}{2}}$

$= 2^{\frac{3}{2}}$

$\log_2 0,0625 = \boxed{-4}$

$\boxed{\frac{1}{16}} = \frac{1}{2^4} = 2^{\boxed{-4}}$

$\log_2 0.0625 \qquad \boxed{-4}$ W.A.

DIGITRON $\ln(e) \qquad \log(\omega)$

$\log_a b = \frac{\log_c b}{\log_c a} \qquad \log_2 0.0625 = \frac{\ln 0.0625}{\ln 2}$

?

$\log_9 81 = 2$ $\quad 9^2 = 81$

$\log_3 81 = 4$ $\quad 3^4 = \overbrace{3 \cdot 3 \cdot 3 \cdot 3}^{4} = 81$

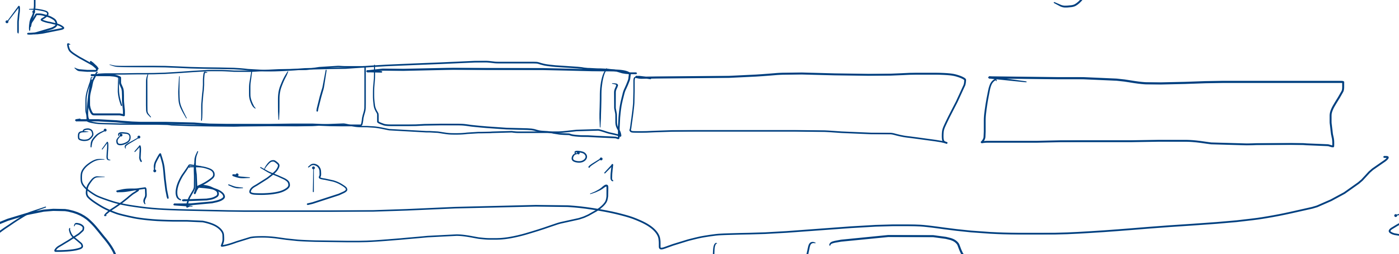
$\log_3 9 = 2$ $\quad \boxed{\log_{10} \underbrace{1000000}_{10^6} = \boxed{6}}$

$y = \log_{\underset{e}{10}} x$

$\log_9 \frac{1}{81} = -2$ $\quad 9^{-2} = \frac{1}{9^2}$

$\log_9 \frac{1}{3} = -\frac{1}{2}$ $\quad 9^{-\frac{1}{2}} = 3^{-1} = \frac{1}{3}$

$\log_3 \frac{1}{9} = -2$ $\quad 3^{\boxed{-2}} = \frac{1}{9}$



2^8 $\quad 2^8 - 1$

$0 - 255$

$2^{16} = 65536$

$32b = \boxed{4B}$

$KB = 1024B$

192.168.0.1

IPV4

275

12.1.3

2^{32} $\quad \cancel{2^{64}}$

2^{64} KOLIKO IMA CIFARA?

IPV6

2^{48}

INTERNET CHINA

IPV6

$\lfloor \log_{10} x \rfloor + 1 = \#CIFARA$

$$\left\lfloor \log_{10} 2^{\boxed{\cancel{32}\ 64}} \right\rfloor + 1 =$$

$$= \left\lfloor \cancel{32}\ 64 \cdot \log_{10} 2 \right\rfloor + 1 =$$

$$0.30103$$

$$= \left\lfloor \underline{\cancel{32}\ 64} \cdot \underline{0,30103} \right\rfloor + 1 =$$

$$\left\lfloor \cancel{9.7}\ 19.6 \right\rfloor + 1 = \underline{\cancel{1}\,2\,0}$$

$$\approx \underbrace{\boxed{\cancel{1}\ 2}\ 000\ 000\ 000}$$

~~IPV6~~

~~1995~~

28

9.

$a: \mathbb{N} \to \mathbb{R}$ $\quad a(n) = a_n = 2^{-(n-1)}$

$a_1, a_2, a_3, \ldots$ $\quad 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$

$\lim_{n\to\infty} 2^{-n+1} = \lim_{n\to\infty} 2 \cdot 2^{-n} = 2 \cdot \lim_{n\to\infty} \left(\frac{1}{2}\right)^n = 2 \cdot 0 = 0$

$(n \to \infty) \to 0$

Granična vrednost niza

$\lim_{n\to\infty} \frac{1}{n} = 0$ $\quad A$

$$A = \lim_{n\to\infty} a_n$$

a_n KONVERGIRA

$$\Leftrightarrow$$

$$\forall \varepsilon > 0\,, \exists n_0 \in \mathbb{N}\,, \forall n \geq n_0\,, |A - a_n| < \varepsilon$$

$|0 - \frac{1}{n}| < \varepsilon$ $\quad n_0(\varepsilon)$ $\quad n_0 = [\frac{1}{\varepsilon}] + 1$ $\quad \frac{1}{n} < \varepsilon$ $\quad n > \frac{1}{\varepsilon}$

Određeni oblici

$\infty \cdot \infty = \infty\,,\quad \infty + \infty = \infty\,,\quad \infty^\infty = \infty\,,$

$\frac{1}{\pm 0} = \pm\infty\,,\quad \frac{1}{\pm\infty} = 0\,,\quad \frac{0}{\pm\infty} = 0\,,$

$0^\infty = 0\,,\quad \frac{\infty}{\pm 0} = \pm\infty\,.$

$\frac{1}{\infty} = 0$

$-\infty, +\infty$ $\quad 0$?

Neodređeni oblici

$\infty - \infty\,,\; 0 \cdot \infty\,,\; \frac{0}{0}\,,\; \frac{\infty}{\infty}\,,\; 1^\infty\,,\; 0^0\,,\; \infty^0\,.$

Izračunati:

1. $\lim_{n\to\infty} \frac{n+1}{\sqrt{n^3}} = \lim_{n\to\infty} \left(\frac{n}{n^{3/2}} + \frac{1}{n^{3/2}}\right) = \lim_{n\to\infty} \left(\frac{1}{n^{1/2}} + \frac{1}{n^{3/2}}\right) = 0$

2. $\lim_{n\to\infty} \frac{n^2+3n}{n-2^n} = -\lim \frac{n^2}{+2^n} = 0$

3. $\lim_{n\to\infty} \frac{n^2+3^n}{n-2^n} \cdot \frac{3^{-n}}{3^{-n}} = \lim_{n\to\infty} \frac{\frac{n^2}{3^n} + 1}{\frac{n}{3^n} - \left(\frac{2}{3}\right)^n} = \frac{0+1}{0-0} = \frac{1}{-0} = -\infty$

4. $\lim_{n\to\infty} \frac{1.1^n}{100^n}$

5. $\lim_{n\to\infty} \frac{\sqrt{n^2+n+1} - \sqrt{n^2+3n-1}}{n+3}$

Važne granične vrednosti

1. $\lim_{n\to\infty} \frac{1}{n^\alpha} = \begin{cases} 0, & \alpha > 0 \\ 1, & \alpha = 0 \\ +\infty, & \alpha < 0 \end{cases}$

$1, 1, 1, \ldots$ $\quad \frac{1}{n^\alpha} = n^{-\alpha} \to \infty$

2. $\lim_{n\to\infty} q^n = \begin{cases} 0, & |q| < 1 \\ 1, & q = 1 \\ +\infty, & q > 1 \end{cases}$

$-2 \quad -1 \quad 0 \quad 1$

DIVERGIRA

3. $\lim_{\substack{n\to\infty \\ f(n)\to\pm\infty}} \left(1 + \frac{1}{f(n)}\right)^{g(n)} = e^{\lim_{n\to\infty} \frac{g(n)}{f(n)}}$

$\lim_{n\to\infty} (-2)^n = ?$ NE POSTOJI $\quad -2, 4, -8, 16, \ldots$

4. $\lim_{n\to\infty} \frac{n^\alpha}{a^n} = 0$, za $a > 1$

5. $\lim_{n\to\infty} (q^n - r^n) = +\infty$, za $q > r > 1$

6. $\lim_{n\to\infty} (n^\alpha - n^\beta) = +\infty$, za $\alpha > \beta > 0$

SVI NIZOVI

KONVERGENTNI

DIVERGENTNI

IMAJU GR. VR.

$\lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^{n} = e^{1}$

$\lim_{n\to\infty} \frac{n^{10}}{1{,}01^n} = 0$

1000000 $\quad 1{,}01$

$n = 2 \quad 1{,}0201 \quad 1{,}02 \quad 1{,}04$

1000000

Granična vrednost funkcije

$$\lim_{x\to a} f(x) = l \Leftrightarrow$$

$$\forall \epsilon > 0\ \exists \delta > 0\ (0 < |x-a| < \delta \Rightarrow |f(x) - l| < \epsilon)$$

Ako granične vrednosti postoje, važi:

$$\lim_{x\to a}(\alpha f(x) + \beta g(x)) = \alpha \lim_{x\to a} f(x) + \beta \lim_{x\to a} g(x)$$

$$\lim_{x\to a}(f(x)\cdot g(x)) = \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x)$$

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \frac{\lim\limits_{x\to a} f(x)}{\lim\limits_{x\to a} g(x),} \quad \lim_{x\to a} g(x) \neq 0$$

$$\lim_{x\to a} h(f(x)) = h\left(\lim_{x\to a} f(x)\right),\ h \text{ neprekidno.}$$

Važne granične vrednosti

1. $\lim\limits_{x\to 0} \frac{\sin x}{x} = 1$
2. $\lim\limits_{x\to \pm\infty} \left(1 + \frac{1}{x}\right)^x = e$
3. $\lim\limits_{x\to 0} (1+x)^{\frac{1}{x}} = e$
4. $\lim\limits_{x\to 0} \frac{e^x - 1}{x} = 1$
5. $\lim\limits_{x\to 0} \frac{\ln(1+x)}{x} = 1$
6. $\lim\limits_{x\to \infty} \arctan x = \pi/2$

Izračunati:

1. $\lim\limits_{x\to\infty} \frac{x^2+3x}{2x^3-4x}$,
2. $\lim\limits_{x\to\infty} \frac{x^2+3x}{2x^2-4x}$,
3. $\lim\limits_{x\to\infty} \frac{x^3+3x}{2x^2-4x}$,
4. $\lim\limits_{x\to 0} \frac{x^2+3x}{2x^3-4x}$,
5. $\lim\limits_{x\to 0} \frac{x^2+3x}{2x^2-4x}$,
6. $\lim\limits_{x\to 0} \frac{x^3+3}{2x^2-4x}$.

Granična vrednost funkcije

$$\lim_{x\to a} f(x) = l \Leftrightarrow$$

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Ako granične vrednosti postoje, važi:

$$\lim_{x\to a}(\alpha f(x) + \beta g(x)) = \alpha \lim_{x\to a} f(x) + \beta \lim_{x\to a} g(x)$$

$$\lim_{x\to a}(f(x)\cdot g(x)) = \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x)$$

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x),} \quad \lim_{x\to a} g(x) \neq 0$$

$$\lim_{x\to a} h(f(x)) = h\left(\lim_{x\to a} f(x)\right), \; h \text{ neprekidno.}$$

Važne granične vrednosti

1. $\lim_{x\to 0} \frac{\sin x}{x} = 1$
2. $\lim_{x\to \pm\infty} \left(1 + \frac{1}{x}\right)^x = e$
3. $\lim_{x\to 0} (1+x)^{\frac{1}{x}} = e$
4. $\lim_{x\to 0} \frac{e^x - 1}{x} = 1$
5. $\lim_{x\to 0} \frac{\ln(1+x)}{x} = 1$
6. $\lim_{x\to \infty} \arctan x = \pi/2$

Izračunati:

1. $\lim_{x\to 2} \frac{x^2-4}{x^2-5x+6}$,

2. $\lim_{x\to 4^+} \frac{\sqrt{x}-2}{\sqrt{x-4}}$,

3. $\lim_{x\to 0} \frac{\sqrt{1+x}-1}{x}$.

4. $\lim_{x\to 0} \frac{\tan x}{x}$.

5. $\lim_{x\to 0} \frac{1-\cos x}{x^2}$.

1. Koristeći kalkulator rešiti jednačinu $1.16\,x^2 - 0.32\,x - 1.361364 = 0$.

2. Koristeći kalkulator izračunati graničnu vrednost $\lim\limits_{n\to\infty}(\sqrt{n^2+4n+1}-\sqrt{n^2+n})$.

3. Koristeći kalkulator izračunati graničnu vrednost $\lim\limits_{x\to 0}\dfrac{1-\cos x}{x^2}$.

Izvod funkcije

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

Ako funkcija ima prvi izvod u tački x, kažemo da je **diferencijabilna** u toj tački.

Osobine izvoda

1. $(\alpha f(x) + \beta g(x))' = \alpha f'(x) + \beta g'(x)$
2. $(f(x) \cdot g(x))' = f'(x)g(x) + f(x)g'(x)$
3. $\left(\dfrac{f(x)}{g(x)}\right)' = \dfrac{f'(x)g(x) - f(x)g'(x)}{g^2(x)}$
4. $(f(g(x)))' = f'(g(x))g'(x)$

Tablica izvoda

i) $c' = 0$

ii) $(x^n)' = nx^{n-1}$, $n \neq 0$

iii) $(\log_a x)' = \dfrac{1}{x \ln a}$

iv) $(a^x)' = a^x \ln a$

v) $(\sin x)' = \cos x$

vi) $(\cos x)' = -\sin x$

vii) $(\arcsin x)' = \dfrac{1}{\sqrt{1-x^2}}$

viii) $(\operatorname{arctg} x)' = \dfrac{1}{1+x^2}$

Pokazati po definiciji da je $(x^2 - x + 1)' = 2x - 1$.

$$\lim_{\Delta x \to 0} \frac{(x+\Delta x)^2 - (x+\Delta x) + 1 - (x^2 - x + 1)}{\Delta x} =$$

$$= \lim_{\Delta x \to 0} \frac{x^2 + 2x\Delta x + \Delta x^2 - x - \Delta x + 1 - x^2 + x - 1}{\Delta x} =$$

$$= \lim_{\Delta x \to 0} \left(\frac{\Delta x(2x-1)}{\Delta x} + \Delta x \right) = 2x - 1$$

Naći po definiciji izvod funkcije $f(x) = \sqrt{x}$

Izvod funkcije

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

Ako funkcija ima prvi izvod u tački x, kažemo da je **diferencijabilna** u toj tački.

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2. $(f(x) \cdot g(x))' = f'(x)g(x) + f(x)g'(x)$
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viii) $(\operatorname{arctg} x)' = \frac{1}{1 + x^2}$

1. $y = x\sqrt{x} - 3\sin x - \frac{1}{x} + \ln x^2, \quad y' =$

2. $y = (x^2 + x)^2 - e^{3+x} + \sqrt{2x}, \quad y' =$

3. $y = \frac{x}{\sqrt{x}} + 2\cos x + \ln(2x), \quad y' =$

4. $y = \frac{1}{x\sqrt{x}} - \frac{1}{e^{-x}} + \log_2 x, \quad y' =$