FTN, EET **Operaciona istraživanja - kol. 1** 20. IV 2023.

Snabdevač studentskog restorana treba da isporuči 1000 litara egzotika. Egzotik se spravlja mešanjem četiri vrste napitaka čije cene i karakteristike su date u tabeli.

	A	B	C	D
Orange juice [%]	80	60	0	0
Grapefruit juice [%]	0	10	100	0
cena [RSD/l]	40	45	80	5

U egzotiku treba da bude barem 30% đusa od narandže i barem 30% đusa od grejpfruta.

1. Postaviti problem linearnog programiranja kojim se nalazi najjeftinija smesa koja zadovoljava uslove. (Jasno napisati značenje uvedenih veličina.)
2. Dovesti postavljeni problem na oblik sa jednakostima.
3. Rešiti postavljeni problem Dualnim simplex algoritmom.
4. Napisati optimalni rečnik.
5. U kom opsegu može da se promeni cena napitka A tako da rešenje ostane optimalno?

Rešenja

1. Uvešćemo veličine:
   ζ = cena isporuke egzotika [RSD]
   x_1 = količina napitka A u isporuci egzotika [l]
   x_2 = količina napitka B u isporuci egzotika [l]
   x_3 = količina napitka C u isporuci egzotika [l]
   x_4 = količina napitka D u isporuci egzotika [l]

$$
\begin{array}{rcrcrcrcrcl}
\zeta & = & 40x_1 & + & 45x_2 & + & 80x_3 & + & 5x_4 & \rightarrow & \min \\
 & & x_1 & + & x_2 & + & x_3 & + & x_4 & \geq & 1000 \\
 & & 0.80x_1 & + & 0.60x_2 & & & & & \geq & 300 \\
 & & x_1 & + & 0.10x_2 & + & x_3 & & & \geq & 300 \\
\end{array}
$$
$$
x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0, \quad x_4 \geq 0.
$$

2.

$$
\begin{array}{rcrcrcrcrcrcrcrcl}
-\zeta & = & -40x_1 & - & 45x_2 & - & 80x_3 & - & 5x_4 & & & & & & & \rightarrow & \max \\
 & & -x_1 & - & x_2 & - & x_3 & - & x_4 & + & w_1 & & & & & = & -1000 \\
 & & -0.80x_1 & - & 0.60x_2 & & & & & & & + & w_2 & & & = & -300 \\
 & & -x_1 & - & 0.10x_2 & - & x_3 & & & & & & & + & w_3 & = & -300 \\
\end{array}
$$
$$
x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0, \quad x_4 \geq 0, \quad w_1 \geq 0, \quad w_2 \geq 0, \quad w_3 \geq 0.
$$

3.

0	x_1	x_2	x_3	x_4	w_1	w_2	w_3	
w_1	-1	-1	-1	-1	1	0	0	-1000
w_2	$-4/5$	$-3/5$	0	0	0	1	0	-300
w_3	0	$-1/10$	-1	0	0	0	1	-300
	40	45	80	5	0	0	0	0

1	x_1	x_2	x_3	x_4	w_1	w_2	w_3	
x_4	1	1	1	1	-1	0	0	1000
w_2	$-4/5$	$-3/5$	0	0	0	1	0	-300
w_3	0	$-1/10$	-1	0	0	0	1	-300
	35	40	75	0	5	0	0	-5000

2	x_1	x_2	x_3	x_4	w_1	w_2	w_3	
x_4	0	$1/4$	1	1	-1	$5/4$	0	625
x_1	1	$3/4$	0	0	0	$-5/4$	0	375
w_3	0	$-1/10$	-1	0	0	0	1	-300
	0	$55/4$	75	0	5	$175/4$	0	-18125

3	x_1	x_2	x_3	x_4	w_1	w_2	w_3	
x_4	0	$3/20$	0	1	-1	$5/4$	1	325
x_1	1	$3/4$	0	0	0	$-5/4$	0	375
x_3	0	$1/10$	1	0	0	0	-1	300
	0	$25/4$	0	0	5	$175/4$	75	-40625

$$x_1 = 375,\ x_2 = 0,\ x_3 = 300,\ x_4 = 325,\ \zeta = 40625.$$

4.

$$\begin{array}{rcrcrcrcrcr}
-\zeta & = & -40625 & - & \frac{25}{4}x_2 & - & 5w_1 & - & \frac{175}{4}w_2 & - & 75w_3 \\
\hline
x_4 & = & 325 & - & \frac{3}{20}x_2 & + & w_1 & - & \frac{5}{4}w_2 & - & w_3 \\
x_1 & = & 375 & - & \frac{3}{4}x_2 & & & + & \frac{5}{4}w_2 & & \\
x_3 & = & 300 & - & \frac{1}{10}x_2 & & & & & + & w_3
\end{array}$$

5.

$$c := c + t\Delta c,\ \Delta c = [1,0,0,0,0,0,0]^T,\ \Delta c_B = [0,1,0]^T,\ \Delta c_N = [0,0,0,0]^T.$$

$$z_N^* = [\frac{25}{4}, 5, \frac{175}{4}, 75]^T,\ \Delta z_N = (B^{-1}N)^T \Delta c_B - \Delta c_N = \left[\frac{3}{4}, 0, -\frac{5}{4}, 0\right]^T.$$

$$z_N^* := z_N^* + t\Delta z_N \geq [0,0,0,0]^T \ \Leftrightarrow\ t \in \left[-\frac{25}{3}, 35\right] \ \Leftrightarrow$$

$$\Leftrightarrow\ -c_1 \in \left[-40 - \frac{25}{3}, -40 + 35\right] \ \Leftrightarrow\ c_1 \in \left[5, \frac{145}{3}\right].$$