Primar - standardni oblik sa $\leq$

$c^T x \to \max$

$Ax \leq b$

$x \geq 0$

Dual - standardni oblik sa $\geq$

$b^T y \to \min$

$A^T y \geq c$

$y \geq 0$

Primar - standardni oblik sa $=$

$c^T x \to \max$

$Ax + w = b$

$x \geq 0, w \geq 0$

Dual - standardni oblik sa $=$

$b^T y \to \min$

$A^T y - z = c$

$y \geq 0, z \geq 0$

Formati matrica: $A_{m\times n},\ b_{m\times 1},\ c_{n\times 1},\ x_{n\times 1},\ w_{m\times 1},\ y_{m\times 1},\ z_{n\times 1}$.

Slaba teorema dualnosti: za dopustive vrednosti primara x i duala y: $c^T x \leq b^T y$.

Jaka teorema dualnosti: za optimalna rešenja primara x i duala y: $c^T x = b^T y$.

Complementary slackness: dopustiva rešenja x i w za primar i dopustiva y i z za odgovarajući dual su optimalni akko $x_j z_j = 0, j = 1, \dots n$, $y_i w_i = 0, i = 1, \dots m$.

Dat je problem linearnog programiranja

$$\zeta = 6x_1 + 5x_2 \to \max$$

$$\begin{array}{rcrcr} -2x_1 & + & x_2 & \leq & 1 \\ -1x_1 & + & 2x_2 & \leq & 5 \\ x_1 & + & x_2 & \leq & 7 \\ 2x_1 & + & x_2 & \leq & 11 \\ x_1 & - & 2x_2 & \leq & 3 \end{array}$$

$$x_1 \geq 0, \quad x_2 \geq 0$$

Od datog problema napraviti standardni oblik sa $=$

$$\zeta = 6x_1 + 5x_2 \to \max$$

$$\begin{array}{rcrcccccccccr} -2x_1 & + & x_2 & + & w_1 & & & & & & & = & 1 \\ -1x_1 & + & 2x_2 & & & + & w_2 & & & & & = & 5 \\ x_1 & + & x_2 & & & & & + & w_3 & & & = & 7 \\ 2x_1 & + & x_2 & & & & & & & + & w_4 & = & 11 \\ x_1 & - & 2x_2 & & & & & & & & + w_5 & = & 3 \end{array}$$

$$x_1 \geq 0, \quad x_2 \geq 0, \quad w_1 \geq 0, \quad w_2 \geq 0, \quad w_3 \geq 0, \quad w_4 \geq 0, \quad w_5 \geq 0$$

Za dati problem postaviti dual, rešiti primar i dual

Dual

$$\xi = y_1 + 5y_2 + 7y_3 + 11y_4 + 3y_5 \to \min$$

$$\begin{array}{rcrcrcrcrcr} -2y_1 & - & y_2 & + & y_3 & + & 2y_4 & + & y_5 & \geq & 6 \\ y_1 & + & 2y_2 & + & y_3 & + & y_4 & - & 2y_5 & \geq & 5 \end{array}$$

$$y_1 \geq 0 \quad y_2 \geq 0 \quad y_3 \geq 0 \quad y_4 \geq 0 \quad y_5 \geq 0$$

Rešenje primara

0	x_1	x_2	w_1	w_2	w_3	w_4	w_5	
w_1	-2	1	1	0	0	0	0	1
w_2	-1	2	0	1	0	0	0	5
w_3	1	1	0	0	1	0	0	7
w_4	2	1	0	0	0	1	0	11
w_5	1	-2	0	0	0	0	1	3
	-6	-5	0	0	0	0	0	0

1	x_1	x_2	w_1	w_2	w_3	w_4	w_5	
w_1	0	-3	1	0	0	0	2	7
w_2	0	0	0	1	0	0	1	8
w_3	0	3	0	0	1	0	-1	4
w_4	0	5	0	0	0	1	-2	5
x_1	1	-2	0	0	0	0	1	3
	0	-17	0	0	0	0	6	18

2	x_1	x_2	w_1	w_2	w_3	w_4	w_5	
w_1	0	0	1	0	0	$3/5$	$4/5$	10
w_2	0	0	0	1	0	0	1	8
w_3	0	0	0	0	1	$-3/5$	$1/5$	1
x_2	0	1	0	0	0	$1/5$	$-2/5$	1
x_1	1	0	0	0	0	$2/5$	$1/5$	5
	0	0	0	0	0	$17/5$	$-4/5$	35

3	x_1	x_2	w_1	w_2	w_3	w_4	w_5	
w_1	0	0	1	0	-4	3	0	6
w_2	0	0	0	1	-5	3	0	3
w_5	0	0	0	0	5	-3	1	5
x_2	0	1	0	0	2	-1	0	3
x_1	1	0	0	0	-1	1	0	4
	0	0	0	0	4	1	0	39

$$x = [4; 3], w = [6; 3; 0; 0; 5]$$

$$\zeta = 39$$

Rešenje duala očitano iz optimalne tabele primara

$$y = [0; 0; 4; 1; 0], z = [0; 0]$$

$$\xi = 39$$

Uputstvo: dualne promenljive očitavamo: y ispod w, z ispod x

Drugi način: rešićemo dual i očitati rešenje primara

-1	y_1	y_2	y_3	y_4	y_5	z_1	z_2	y_0	
z_1	2	1	-1	-2	-1	1	0	-1	-6
z_2	-1	-2	-1	-1	2	0	1	-1	-5
	1	5	7	11	3	0	0	0	0
	0	0	0	0	0	0	0	1	0

0	y_1	y_2	y_3	y_4	y_5	z_1	z_2	y_0	
y_0	-2	-1	1	2	1	-1	0	1	6
z_2	-3	-3	0	1	3	-1	1	0	1
	1	5	7	11	3	0	0	0	0
	2	1	-1	-2	-1	1	0	0	-6

1	y_1	y_2	y_3	y_4	y_5	z_1	z_2	y_0	
y_0	4	5	1	0	-5	1	-2	1	4
y_4	-3	-3	0	1	3	-1	1	0	1
	34	38	7	0	-30	11	-11	0	-11
	-4	-5	-1	0	5	-1	2	0	-4

2	y_1	y_2	y_3	y_4	y_5	z_1	z_2	
y_2	$4/5$	1	$1/5$	0	-1	$1/5$	$-2/5$	$4/5$
y_4	$-3/5$	0	$3/5$	1	0	$-2/5$	$-1/5$	$17/5$
	$18/5$	0	$-3/5$	0	8	$17/5$	$21/5$	$-207/5$

3	y_1	y_2	y_3	y_4	y_5	z_1	z_2	
y_3	4	5	1	0	-5	1	-2	4
y_4	-3	-3	0	1	3	-1	1	1
	6	3	0	0	5	4	3	-39

$y = [0; 0; 4; 1; 0], z = [0; 0], \xi = 39$

Rešenje primara očitano iz optimalne tabele duala

$x = [4; 3], w = [6; 3; 0; 0; 5], \zeta = 39$

Problem prodavnice zdrave hrane

Prodavnica zdrave hrane pravi smesu dve vrste mizli. U Sport mizlima ima 20% pšeničnih pahuljica, a u Tropic mizlima ima 32% pšeničnih pahuljica. Tropic mizle koštaju 60 pfeniga kg a Sport mizle 80 pfeniga kg. Koliko kojih mizli treba staviti u 1 kg smese pa da se količina pšeničnih pahuljica održi do 25%, a da smesa bude što jeftinija?

Uvodimo veličine x_1 = količina Sport mizli u 1 kg smese, x_2 = količina Tropic mizli u 1 kg smese, ζ = cena 1 kg smese.

$$\zeta = 80x_1 + 60x_2 \to \min$$

$$\begin{array}{rcrcr} 0.20x_1 & + & 0.32x_2 & \leq & 0.25 \\ x_1 & + & x_2 & = & 1 \end{array}$$

$$x_1 \geq 0, \quad x_2 \geq 0$$

Da bi dobili standardni oblik sa $\leq$, jednakost $x_1 + x_2 = 1$ razdvajamo u dve nejednakosti: $x_1 + x_2 \leq 1$ i $x_1 + x_2 \geq 1$, poslednju množimo sa (-1) i ona postaje $-x_1 - x_2 \leq -1$.

-1	x_1	x_2	w_1	w_2	w_3	x_0	
w_1	$1/5$	$8/25$	1	0	0	-1	$1/4$
w_2	1	1	0	1	0	-1	1
w_3	-1	-1	0	0	1	-1	-1
	80	60	0	0	0	0	0
	0	0	0	0	0	1	0

0	x_1	x_2	w_1	w_2	w_3	x_0	
w_1	$6/5$	$33/25$	1	0	-1	0	$5/4$
w_2	2	2	0	1	-1	0	2
x_0	1	1	0	0	-1	1	1
	80	60	0	0	0	0	0
	-1	-1	0	0	1	0	-1

1	x_1	x_2	w_1	w_2	w_3	
w_1	0	$3/25$	1	0	$1/5$	$1/20$
w_2	0	0	0	1	1	0
x_1	1	1	0	0	-1	1
	0	-20	0	0	80	-80

2	x_1	x_2	w_1	w_2	w_3	
x_2	0	1	$25/3$	0	$5/3$	$5/12$
w_2	0	0	0	1	1	0
x_1	1	0	$-25/3$	0	$-8/3$	$7/12$
	0	0	$500/3$	0	$340/3$	$-215/3$

$$x = [7/12; 5/12], w = [0; 0; 0], \zeta = 215/3 = 71 + 2/3$$

Rešenje duala:

$$y = [500/3; 0; 340/3], z = [0; 0], \xi = 71 + 2/3$$

Rešićemo isti problem na drugi način, preko duala

Primar

$$-\zeta = -80x_1 - 60x_2 \to \max$$

$$\begin{array}{rcrcr} 0.20x_1 & + & 0.32x_2 & \leq & 0.25 \\ x_1 & + & x_2 & = & 1 \end{array}$$

$$x_1 \geq 0, \quad x_2 \geq 0$$

Dual

$$-\xi = 0.25y_1 + y_2 \to \min$$

$$\begin{array}{rcrcr} 0.20y_1 & + & y_2 & \geq & -80 \\ 0.32y_1 & + & y_2 & \geq & -60 \end{array}$$

$$y_1 \geq 0$$

Za promenljivu y_2 se ne zahteva nenegativnost, zato uvodimo smenu $y_2 = y_2' - y_2''$, uz uslove $y_2' \geq 0$, $y_2'' \geq 0$.

0	y_1	y_2'	y_2''	z_1	z_2	
z_1	$-1/5$	-1	1	1	0	80
z_2	$-8/25$	-1	1	0	1	60
	$1/4$	1	-1	0	0	0

1	y_1	y_2'	y_2''	z_1	z_2	
z_1	$3/25$	0	0	1	-1	20
y_2''	$-8/25$	-1	1	0	1	60
	$-7/100$	0	0	0	1	60

2	y_1	y_2'	y_2''	z_1	z_2	
y_1	1	0	0	$25/3$	$-25/3$	$500/3$
y_2''	0	-1	1	$8/3$	$-5/3$	$340/3$
	0	0	0	$7/12$	$5/12$	$215/3$

$$y = [y_1; y_2'; y_2''] = [500/3; 0; 340/3], y_2 = y_2' - y_2'' = -340/3, z = [0; 0]$$

$$\xi = 71 + 2/3$$

Očitavamo i rešenje primara, koji je dual od duala:

$$x = [7/12; 5/12], w = [0; 0; 0], \zeta = 71 + 2/3$$

Za problem linearnog programiranja postaviti dual, rešiti primar i dual

$$\zeta = 3x_1 + 4x_2 - x_3 \to \max$$

$$\begin{array}{rcrcrcr} 4x_1 & - & x_2 & - & 2x_3 & \geq & 4 \\ 3x_1 & + & 5x_2 & & & \leq & 15 \\ x_1 & + & x_2 & - & x_3 & \leq & 6 \end{array}$$

$$x_1 \geq 0, \quad x_2 \geq 0$$

$$\xi = -4y_1 + 15y_2 + 6y_3 \to \min$$

$$\begin{array}{rcrcrcr} -4y_1 & + & 3y_2 & + & y_3 & \geq & 3 \\ y_1 & + & 5y_2 & + & y_3 & \geq & 4 \\ 2y_1 & & & - & y_3 & = & -1 \end{array}$$

$$y_1 \geq 0, \quad y_2 \geq 0, \quad y_3 \geq 0$$

Uvodimo smenu $x_3 = x_3' - x_3''$, dodajemo $x_3' \geq 0$ i $x_3'' \geq 0$.

-1	x_1	x_2	x_3'	x_3''	w_1	w_2	w_3	x_0	
w_1	-4	1	2	-2	1	0	0	-1	-4
w_2	3	5	0	0	0	1	0	-1	15
w_3	1	1	-1	1	0	0	1	-1	6
	-3	-4	1	-1	0	0	0	0	0
	0	0	0	0	0	0	0	1	0

0	x_1	x_2	x_3'	x_3''	w_1	w_2	w_3	x_0	
x_0	4	−1	−2	2	−1	0	0	1	4
w_2	7	4	−2	2	−1	1	0	0	19
w_3	5	0	−3	3	−1	0	1	0	10
	−3	−4	1	−1	0	0	0	0	0
	−4	1	2	−2	1	0	0	0	−4

1	x_1	x_2	x_3'	x_3''	w_1	w_2	w_3	
x_1	1	−1/4	−1/2	1/2	−1/4	0	0	1
w_2	0	23/4	3/2	−3/2	3/4	1	0	12
w_3	0	5/4	−1/2	1/2	1/4	0	1	5
	0	−19/4	−1/2	1/2	−3/4	0	0	3

2	x_1	x_2	x_3'	x_3''	w_1	w_2	w_3	
x_1	1	0	−10/23	10/23	−5/23	1/23	0	35/23
x_2	0	1	6/23	−6/23	3/23	4/23	0	48/23
w_3	0	0	−19/23	19/23	2/23	−5/23	1	55/23
	0	0	17/23	−17/23	−3/23	19/23	0	297/23

3	x_1	x_2	x_3'	x_3''	w_1	w_2	w_3	
x_1	1	0	0	0	−5/19	3/19	−10/19	5/19
x_2	0	1	0	0	3/19	2/19	6/19	54/19
x_3''	0	0	−1	1	2/19	−5/19	23/19	55/19
	0	0	0	0	−1/19	12/19	17/19	286/19

4	x_1	x_2	x_3'	x_3''	w_1	w_2	w_3	
x_1	1	5/3	0	0	0	1/3	0	5
w_1	0	19/3	0	0	1	2/3	2	18
x_3''	0	−2/3	−1	1	0	−1/3	1	1
	0	1/3	0	0	0	2/3	1	16

$$x = [x_1, x_2, x_3', x_3''] = [5; 0; 0; 1], x_3 = x_3' - x_3'' = -1, w = [18; 0; 0], \zeta = 16$$

Rešenje duala očitavamo iz optimalne tabele: $y = [0; 2/3; 1], z = [0; 1/3; 0; 0], \xi = 16$.

Rešiti problem linearnog programiranja

$$\zeta = 10x_1 - 57x_2 - 9x_3 - 24x_4 \to \max$$

$$\begin{array}{rcrcrcrcl} 1/2x_1 & - & 11/2x_2 & - & 5/2x_3 & + & 9x_4 & \leq & 0 \\ 1/2x_1 & - & 3/2x_2 & - & 1/2 & + & x_4 & \leq & 0 \\ x_1 & & & & & & & \leq & 1 \end{array}$$

$$x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0, \quad x_4 \geq 0$$

U ovom zadatku imamo problem degeneracije, moguće je da se desi ciklično kruženje od tabele do tabele. Da bi se to sprečilo, pri izboru radne kolone i pivota ćemo koristiti pravilo Blanda.

0	x_1	x_2	x_3	x_4	w_1	w_2	w_3	
w_1	$1/2$	$-11/2$	$-5/2$	9	1	0	0	0
w_2	$1/2$	$-3/2$	$-1/2$	1	0	1	0	0
w_3	1	0	0	0	0	0	1	1
	-10	57	9	24	0	0	0	0

1	x_1	x_2	x_3	x_4	w_1	w_2	w_3	
x_1	1	-11	-5	18	2	0	0	0
w_2	0	4	2	-8	-1	1	0	0
w_3	0	11	5	-18	-2	0	1	1
	0	-53	-41	204	20	0	0	0

2	x_1	x_2	x_3	x_4	w_1	w_2	w_3	
x_1	1	0	$1/2$	-4	$-3/4$	$11/4$	0	0
x_2	0	1	$1/2$	-2	$-1/4$	$1/4$	0	0
w_3	0	0	$-1/2$	4	$3/4$	$-11/4$	1	1
	0	0	$-29/2$	98	$27/4$	$53/4$	0	0

3	x_1	x_2	x_3	x_4	w_1	w_2	w_3	
x_3	2	0	1	-8	$-3/2$	$11/2$	0	0
x_2	-1	1	0	2	$1/2$	$-5/2$	0	0
w_3	1	0	0	0	0	0	1	1
	29	0	0	-18	-15	93	0	0

4	x_1	x_2	x_3	x_4	w_1	w_2	w_3	
x_3	-2	4	1	0	$1/2$	$-9/2$	0	0
x_4	$-1/2$	$1/2$	0	1	$1/4$	$-5/4$	0	0
w_3	1	0	0	0	0	0	1	1
	20	9	0	0	$-21/2$	$141/2$	0	0

5	x_1	x_2	x_3	x_4	w_1	w_2	w_3	
w_1	-4	8	2	0	1	-9	0	0
x_4	$1/2$	$-3/2$	$-1/2$	1	0	1	0	0
w_3	1	0	0	0	0	0	1	1
	-22	93	21	0	0	-24	0	0

Biranje radne kolone po pravilu izbora nejnegativnijeg elementa bi nas dovelo u tabelu broj 0. Treba izbeći ciklično kretanje kroz tabele. Zato po pravilu Blanda biramo radnu kolonu x_1.

6	x_1	x_2	x_3	x_4	w_1	w_2	w_3	
w_1	0	-4	-2	8	1	-1	0	0
x_1	1	-3	-1	2	0	2	0	0
w_3	0	3	1	-2	0	-2	1	1
	0	27	-1	44	0	20	0	0

7	x_1	x_2	x_3	x_4	w_1	w_2	w_3	
w_1	0	2	0	4	1	-5	2	2
x_1	1	0	0	0	0	0	1	1
x_3	0	3	1	-2	0	-2	1	1
	0	30	0	42	0	18	1	1

$x = [1; 0; 1; 0], w = [2; 0; 0], \zeta = 1$

Problem snabdevača studentskog restorana

Snabdevač studentskog restorana treba da spremi barem 500 litara egzotika od pet sokova iz skladišta. Egzotik treba da sadrži barem 20% đusa od narandže, 10 % đusa od grejpfruta i 5% đusa od kupine. Koliko kojeg soka treba da smeša u egzotik pa da postigne minimalnu cenu?

sok	orange juice (%)	grejpfrut juice (%)	kupina juice (%)	zaliha (litara)	cena din / lit
1	40	40	0	200	1.50
2	5	10	20	400	0.75
3	100	0	0	100	2.00
4	0	100	0	50	1.75
5	0	0	0	800	0.25

Uvešćemo veličine: x_i = količina soka i u litrama u isporuci egzotika i cena isporuke: ζ u dinarima.

Jedna postavka

$$\zeta = 1.50x_1 + 0.75x_2 + 2.00x_3 + 1.75x_4 + 0.25x_5 \to \min$$

$$x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0, \quad x_4 \geq 0, \quad x_5 \geq 0$$

Ograničenja za količinu isporuke, sadržaj đusa od narandže, đusa od grejpfruta i đusa od kupine:

$$\begin{array}{rcl} x_1 + x_2 + x_3 + x_4 + x_5 & \geq & 500 \\ 0.40x_1 + 0.05x_2 + x_3 & \geq & 0.20\,(x_1 + x_2 + x_3 + x_4 + x_5) \\ 0.40x_1 + 0.10x_2 \qquad + x_4 & \geq & 0.10\,(x_1 + x_2 + x_3 + x_4 + x_5) \\ 0.20x_2 & \geq & 0.05\,(x_1 + x_2 + x_3 + x_4 + x_5) \end{array}$$

Prebacimo sve promenljive na levu stranu i dodajmo ograničenja zaliha:

$$
\begin{array}{rcl}
x_1 + x_2 + x_3 + x_4 + x_5 & \geq & 500 \\
0.20x_1 - 0.15x_2 + 0.80x_3 - 0.20x_4 - 0.20x_5 & \geq & 0 \\
0.30x_1 \qquad\qquad - 0.10x_3 + 0.90x_4 - 0.10x_5 & \geq & 0 \\
-0.05x_1 + 0.15x_2 - 0.05x_3 - 0.05x_4 - 0.05x_5 & \geq & 0 \\
x_1 & \leq & 200 \\
x_2 & \leq & 400 \\
x_3 & \leq & 100 \\
x_4 & \leq & 50 \\
x_5 & \leq & 800
\end{array}
$$

Prelazimo na dual:

$$\xi = 500y_1 - 200y_5 - 400y_6 - 100y_7 - 50y_8 - 800y_9 \to \max$$

$$
\begin{array}{rcl}
y_1 + 0.20y_2 + 0.30y_3 - 0.05y_4 - y_5 & \leq & 1.50 \\
y_1 - 0.15y_2 + 0.15y_4 - y_6 & \leq & 0.75 \\
y_1 + 0.80y_2 - 0.10y_3 - 0.05y_4 - y_7 & \leq & 2.00 \\
y_1 - 0.20y_2 + 0.90y_3 - 0.05y_4 - y_8 & \leq & 1.75 \\
y_1 - 0.20y_2 - 0.10y_3 - 0.05y_4 - y_9 & \leq & 0.25
\end{array}
$$

$$y_1 \geq 0,\ y_2 \geq 0,\ y_3 \geq 0,\ y_4 \geq 0,\ y_5 \geq 0,\ y_6 \geq 0,\ y_7 \geq 0,\ y_8 \geq 0,\ y_9 \geq 0$$

0	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	z_1	z_2	z_3	z_4	z_5	
z_1	1	1/5	3/10	−1/20	−1	0	0	0	0	1	0	0	0	0	3/2
z_2	1	−3/20	0	3/20	0	−1	0	0	0	0	1	0	0	0	3/4
z_3	1	4/5	−1/10	−1/20	0	0	−1	0	0	0	0	1	0	0	2
z_4	1	−1/5	9/10	−1/20	0	0	0	−1	0	0	0	0	1	0	7/4
z_5	1	−1/5	−1/10	−1/20	0	0	0	0	−1	0	0	0	0	1	1/4
	−500	0	0	0	200	400	100	50	800	0	0	0	0	0	0

1	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	z_1	z_2	z_3	z_4	z_5	
z_1	0	2/5	2/5	0	−1	0	0	0	1	1	0	0	0	−1	5/4
z_2	0	1/20	1/10	1/5	0	−1	0	0	1	0	1	0	0	−1	1/2
z_3	0	1	0	0	0	0	−1	0	1	0	0	1	0	−1	7/4
z_4	0	0	1	0	0	0	0	−1	1	0	0	0	1	−1	3/2
y_1	1	−1/5	−1/10	−1/20	0	0	0	0	−1	0	0	0	0	1	1/4
	0	−100	−50	−25	200	400	100	50	300	0	0	0	0	500	125

2	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	z_1	z_2	z_3	z_4	z_5	
z_1	0	0	2/5	0	−1	0	2/5	0	3/5	1	0	−2/5	0	−3/5	11/20
z_2	0	0	1/10	1/5	0	−1	1/20	0	19/20	0	1	−1/20	0	−19/20	7/17
y_2	0	1	0	0	0	0	−1	0	1	0	0	1	0	−1	7/4
z_4	0	0	1	0	0	0	0	−1	1	0	0	0	1	−1	3/2
y_1	1	0	−1/10	−1/20	0	0	−1/5	0	−4/5	0	0	1/5	0	4/5	3/5
	0	0	−50	−25	200	400	0	50	400	0	0	100	0	400	300

3	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	z_1	z_2	z_3	z_4	z_5	
y_3	0	0	1	0	$-5/2$	0	1	0	$3/2$	$5/2$	0	-1	0	$-3/2$	$11/8$
z_2	0	0	0	$1/5$	$1/4$	-1	$-1/20$	0	$4/5$	$-1/4$	1	$1/20$	0	$-4/5$	$11/40$
y_2	0	1	0	0	0	0	-1	0	1	0	0	1	0	-1	$7/4$
z_4	0	0	0	0	$5/2$	0	-1	-1	$-1/2$	$-5/2$	0	1	1	$1/2$	$1/8$
y_1	1	0	0	$-1/20$	$-1/4$	0	$-1/10$	0	$-13/20$	$1/4$	0	$1/10$	0	$13/20$	$14/19$
	0	0	0	-25	75	400	50	50	475	125	0	50	0	325	$1475/4$

4	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	z_1	z_2	z_3	z_4	z_5	
y_3	0	0	1	0	$-5/2$	0	1	0	$3/2$	$5/2$	0	-1	0	$-3/2$	$11/8$
y_4	0	0	0	1	$5/4$	-5	$-1/4$	0	4	$-5/4$	5	$1/4$	0	-4	$11/8$
y_2	0	1	0	0	0	0	-1	0	1	0	0	1	0	-1	$7/4$
z_4	0	0	0	0	$5/2$	0	-1	-1	$-1/2$	$-5/2$	0	1	1	$1/2$	$1/8$
y_1	1	0	0	0	$-3/16$	$-1/4$	$-1/9$	0	$-9/20$	$3/16$	$1/4$	$1/9$	0	$9/20$	$129/160$
	0	0	0	0	$425/4$	275	$175/4$	50	575	$375/4$	125	$225/4$	0	225	$3225/8$

$$y = [129/160; 7/4; 11/8; 11/8; 0; 0; 0; 0; 0], z = [0; 0; 0; 1/8; 0]$$

$$x = [375/4, 125, 225/4, 0, 225], w = [0; 0; 0; 0; 425/4; 275; 175/4; 50; 575; 375/4; 125; 225/4]$$

$$\zeta = 3225/8 = 403.125$$

Treba nam rešenje primara, ono daje recept i cenu najjeftinije smese:

$$x = [93.75; 125; 56.25; 0; 225], \zeta = 403.125$$

Druga postavka

$$\zeta = 1.50x_1 + 0.75x_2 + 2.00x_3 + 1.75x_4 + 0.25x_5 \to \min$$

$$x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0, \quad x_4 \geq 0, \quad x_5 \geq 0$$

Iz tabele sa cenama možemo uočiti da ni jedna sirovina nije besplatna, a da su ograničenja za sadržaj data u procentima. Jasno je da u optimalnom rešenju koje daje minimalnu cenu mora biti količina isporuke tačno 500 litara.
Zaliha soka broj 5 je veća od isporuke ($800 > 500$), to ograničenje možemo ignorisati. Sok broj 4 je 100% kupina đus, njegova zaliha je veća od 5% isporuke ($100 > 0.05 \cdot 500 = 25$), slično za sok broj 3, ograničenja za njihove zalihe možemo ignorisati.
Koristeći ova razmatranja možemo postaviti jednostavnija ograničenja za sadržaj i izbaciti pomenuta ograničenja za zalihe:

$$\begin{array}{rcl}
x_1 + x_2 + x_3 + x_4 + x_5 & \geq & 500 \\
0.40x_1 + 0.05x_2 + x_3 \qquad & \geq & 0.20 \cdot 500 = 100 \\
0.40x_1 + 0.10x_2 \qquad + x_4 & \geq & 0.10 \cdot 500 = 50 \\
0.20x_2 \qquad\qquad & \geq & 0.05 \cdot 500 = 25 \\
x_1 \qquad\qquad\qquad & \leq & 200 \\
x_2 \qquad\qquad & \leq & 400
\end{array}$$

Prelazak na dual daje jednostavniju tabelu nego u prethodnoj postavci. Rešenje je isto za primar, rešenje duala se, naravno, razlikuje.

0	y_1	y_2	y_3	y_4	y_5	y_6	z_1	z_2	z_3	z_4	z_5	
z_1	1	2/5	2/5	0	-1	0	1	0	0	0	0	3/2
z_2	1	1/20	1/10	1/5	0	-1	0	1	0	0	0	3/4
z_3	1	1	0	0	0	0	0	0	1	0	0	2
z_4	1	0	1	0	0	0	0	0	0	1	0	7/4
z_5	1	0	0	0	0	0	0	0	0	0	1	1/4
	-500	-100	-50	-25	200	400	0	0	0	0	0	0

1	y_1	y_2	y_3	y_4	y_5	y_6	z_1	z_2	z_3	z_4	z_5	
z_1	0	2/5	2/5	0	-1	0	1	0	0	0	-1	5/4
z_2	0	1/20	1/10	1/5	0	-1	0	1	0	0	-1	1/2
z_3	0	1	0	0	0	0	0	0	1	0	-1	7/4
z_4	0	0	1	0	0	0	0	0	0	1	-1	3/2
y_1	1	0	0	0	0	0	0	0	0	0	1	1/4
	0	-100	-50	-25	200	400	0	0	0	0	500	125

2	y_1	y_2	y_3	y_4	y_5	y_6	z_1	z_2	z_3	z_4	z_5	
z_1	0	0	2/5	0	-1	0	1	0	$-2/5$	0	$-3/5$	11/20
z_2	0	0	1/10	1/5	0	-1	0	1	$-1/20$	0	$-19/20$	7/17
y_2	0	1	0	0	0	0	0	0	1	0	-1	7/4
z_4	0	0	1	0	0	0	0	0	0	1	-1	3/2
y_1	1	0	0	0	0	0	0	0	0	0	1	1/4
	0	0	-50	-25	200	400	0	0	100	0	400	300

3	y_1	y_2	y_3	y_4	y_5	y_6	z_1	z_2	z_3	z_4	z_5	
y_3	0	0	1	0	$-5/2$	0	5/2	0	-1	0	$-3/2$	11/8
z_2	0	0	0	1/5	1/4	-1	$-1/4$	1	1/20	0	$-4/5$	11/40
y_2	0	1	0	0	0	0	0	0	1	0	-1	7/4
z_4	0	0	0	0	5/2	0	$-5/2$	0	1	1	1/2	1/8
y_1	1	0	0	0	0	0	0	0	0	0	1	1/4
	0	0	0	-25	75	400	125	0	50	0	325	1475/4

4	y_1	y_2	y_3	y_4	y_5	y_6	z_1	z_2	z_3	z_4	z_5	
y_3	0	0	1	0	$-5/2$	0	5/2	0	-1	0	$-3/2$	11/8
y_4	0	0	0	1	5/4	-5	$-5/4$	5	1/4	0	-4	11/8
y_2	0	1	0	0	0	0	0	0	1	0	-1	7/4
z_4	0	0	0	0	5/2	0	$-5/2$	0	1	1	1/2	1/8
y_1	1	0	0	0	0	0	0	0	0	0	1	1/4
	0	0	0	0	425/4	275	375/4	125	225/4	0	225	3225/8

$x = [93.75; 125; 56.25; 0; 225], \zeta = 403.125$

Problem angažovanja u supermarketu

Supermarket pravi raspored radnika na kasi po smenama. Smene traju po 8 sati. Počeci smena i dnevnice (€) su dati u tabeli.

00h	04h	08h	12h	16h	20h
30	30	20	20	25	30

U sledećoj tabeli su date potrebe za radnicima na kasi u periodima dana.

0-4	4-8	8-12	12-16	16-20	20-24
3	2	5	6	7	4

Naći najjeftiniji plan angažovanja radnika na kasi uz zadovoljenost datih potreba.

Uvešćemo veličine: x_i = broj radnika koji počinju i-tu smenu. Promenljive x_i moraju pripadati skupu celih brojeva. Cena dnevnog angažovanja je ζ.

$$\zeta = 30x_1 + 30x_2 + 20x_3 + 20x_4 + 25x_5 + 30x_6 \to \min$$

$$\begin{array}{cccccccccccccc}
x_1 & + & x_2 & & & & & & & & & \geq & 2 \\
 & & x_2 & + & x_3 & & & & & & & \geq & 5 \\
 & & & & x_3 & + & x_4 & & & & & \geq & 6 \\
 & & & & & & x_4 & + & x_5 & & & \geq & 7 \\
 & & & & & & & & x_5 & + & x_6 & \geq & 4 \\
x_1 & & & & & & & & & & x_6 & \geq & 3
\end{array}$$

$$x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0, \quad x_4 \geq 0, \quad x_5 \geq 0, \quad x_6 \geq 0$$

Prelazimo na dual i rešavamo ga simplex metodom.

0	y_1	y_2	y_3	y_4	y_5	y_6	z_1	z_2	z_3	z_4	z_5	z_6	
z_1	1	0	0	0	0	1	1	0	0	0	0	0	30
z_2	1	1	0	0	0	0	0	1	0	0	0	0	30
z_3	0	1	1	0	0	0	0	0	1	0	0	0	20
z_4	0	0	1	1	0	0	0	0	0	1	0	0	20
z_5	0	0	0	1	1	0	0	0	0	0	1	0	25
z_6	0	0	0	0	1	1	0	0	0	0	0	1	30
	-2	-5	-6	-7	-4	-3	0	0	0	0	0	0	0

1	y_1	y_2	y_3	y_4	y_5	y_6	z_1	z_2	z_3	z_4	z_5	z_6	
z_1	1	0	0	0	0	1	1	0	0	0	0	0	30
z_2	1	1	0	0	0	0	0	1	0	0	0	0	30
z_3	0	1	1	0	0	0	0	0	1	0	0	0	20
y_4	0	0	1	1	0	0	0	0	0	1	0	0	20
z_5	0	0	-1	0	1	0	0	0	0	-1	1	0	5
z_6	0	0	0	0	1	1	0	0	0	0	0	1	30
	-2	-5	1	0	-4	-3	0	0	0	7	0	0	140

2	y_1	y_2	y_3	y_4	y_5	y_6	z_1	z_2	z_3	z_4	z_5	z_6	
z_1	1	0	0	0	0	1	1	0	0	0	0	0	30
z_2	1	0	−1	0	0	0	0	1	−1	0	0	0	10
y_2	0	1	1	0	0	0	0	0	1	0	0	0	20
y_4	0	0	1	1	0	0	0	0	0	1	0	0	20
z_5	0	0	−1	0	1	0	0	0	0	−1	1	0	5
z_6	0	0	0	0	1	1	0	0	0	0	0	1	30
	−2	0	6	0	−4	−3	0	0	5	7	0	0	240

3	y_1	y_2	y_3	y_4	y_5	y_6	z_1	z_2	z_3	z_4	z_5	z_6	
z_1	1	0	0	0	0	1	1	0	0	0	0	0	30
z_2	1	0	−1	0	0	0	0	1	−1	0	0	0	10
y_2	0	1	1	0	0	0	0	0	1	0	0	0	20
y_4	0	0	1	1	0	0	0	0	0	1	0	0	20
y_5	0	0	−1	0	1	0	0	0	0	−1	1	0	5
z_6	0	0	1	0	0	1	0	0	0	1	−1	1	25
	−2	0	2	0	0	−3	0	0	5	3	4	0	260

4	y_1	y_2	y_3	y_4	y_5	y_6	z_1	z_2	z_3	z_4	z_5	z_6	
z_1	1	0	−1	0	0	0	1	0	0	−1	1	−1	5
z_2	1	0	−1	0	0	0	0	1	−1	0	0	0	10
y_2	0	1	1	0	0	0	0	0	1	0	0	0	20
y_4	0	0	1	1	0	0	0	0	0	1	0	0	20
y_5	0	0	−1	0	1	0	0	0	0	−1	1	0	5
y_6	0	0	1	0	0	1	0	0	0	1	−1	1	25
	−2	0	5	0	0	0	0	0	5	6	1	3	335

5	y_1	y_2	y_3	y_4	y_5	y_6	z_1	z_2	z_3	z_4	z_5	z_6	
y_1	1	0	−1	0	0	0	1	0	0	−1	1	−1	5
z_2	0	0	0	0	0	0	−1	1	−1	1	−1	1	5
y_2	0	1	1	0	0	0	0	0	1	0	0	0	20
y_4	0	0	1	1	0	0	0	0	0	1	0	0	20
y_5	0	0	−1	0	1	0	0	0	0	−1	1	0	5
y_6	0	0	1	0	0	1	0	0	0	1	−1	1	25
	0	0	3	0	0	0	2	0	5	4	3	1	345

Rešenje duala je: $y = [5; 20; 0; 20; 5; 25], z = [0; 5; 0; 0; 0; 0]$.

$$x = [2; 0; 5; 4; 3; 1], w = [0; 0; 3; 0; 0; 0], \zeta = 345$$

Plan angažovanja po smenama je u x. U trećoj smeni će biti tri radnika viška. Dnevna cena angažovanja radnika je 385€.
Dobili smo celobrojno rešenje.