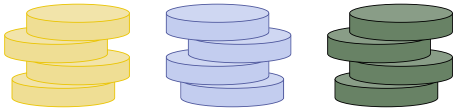
Stacks



The Stack ADT

- Set of objects in which the location an item is inserted and deleted is pre-specified
- Stacks
  - Insert in order
  - Delete most recent item inserted
  - LIFO - last in, first out

The Stack ADT

- Examples of stacks
  - Cafeteria tray dispenser
  - Coin dispenser in your car
  - Balancing braces
  - Recognizing strings in a language
  - Evaluating postfix expressions
  - Converting infix to postfix
  - Undo sequence in a text editor
  - Saving local variables when one function calls another, and this one calls another, and so on.

The Stack ADT

- Main stack operations:
  - push(object ***o***): inserts element ***o***
  - pop(): removes the last inserted element
  - top(): returns a reference to the last inserted element without removing it
- Auxiliary stack operations:
  - size(): returns the number of elements stored
  - isEmpty(): returns true if the stack is empty, else false

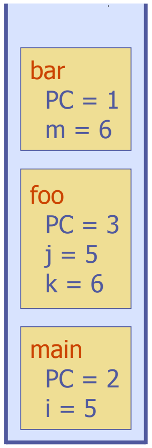
Exceptions

- Attempting the execution of an operation of ADT may sometimes cause an error condition, called an exception
- Exceptions are said to be “thrown” by an operation that cannot be executed
- In the Stack ADT, operations pop and top cannot be performed if the stack is empty
- Attempting the execution of pop or top on an empty stack throws an EmptyStackException

C++ Run-time Stack

- The C++ run-time system keeps track of the chain of active functions with a stack
- When a function is called, the run-time system pushes on the stack a frame containing
  - Local variables and return value
  - Program counter, keeping track of the statement being executed
- When a function returns, its frame is popped from the stack and control is passed to the method on top of the stack

```
main() {
  int i = 5;
  foo(i);
}
foo(int j) {
  int k;
  k = j+1;
  bar(k);
}
bar(int m) {
  func call
}
```



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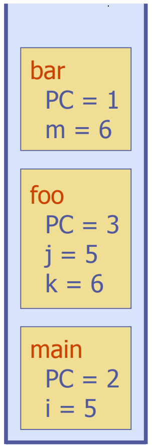
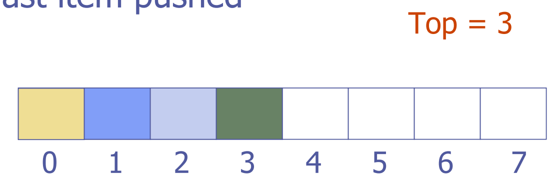


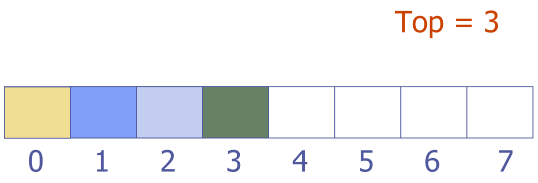
Array-based Stack

- A simple way of implementing the Stack ADT uses an array
- We push (add) elements from left to right
- A variable keeps track of the index of the last item pushed



Array-based Stack

- We pop (remove) elements from right to left



Stack Data Structure

```
class Stack {
private:
          objectType stack[MAX_STACK_SIZE];
          int top;
public:
          functions for stack manipulation
          constructor sets top to -1
};
```

Stack Implementation- Push

- The array storing the stack elements may become full
- A push operation will then throw a FullStackException
  - Limitation of the array-based implementation

```
void push ( objectType o ) {
          if ( top + 1 == MAX_STACK_SIZE )
                    throw FullStackException
          else
                    S[++top] = o;
```

Stack Implementation- Pop

- In class exercise - write pop and getTop functions
  - Array may be empty when pop
  - getTop will return top item/object
  - Operations will may throw an EmptyStackException

Performance and Limitations

- Performance
  - Let n be the number of elements in the stack
  - The space used is $O(n)$
  - Each operation runs in time $O(1)$
- Limitations
  - The maximum size of the stack must be defined *a priori* , and cannot be changed
  - Trying to push a new element into a full stack causes an implementation-specific exception

Stack Application - Infix to Postfix Conversion

- Stack can be used to convert infix mathematical expressions to postfix mathematical expressions

Stack Application - Infix to Postfix Conversion

- Algorithm
  - Process infix expression one item at a time
  - Operand - write to output
  - Operator - pop and write to output until an entry of lower priority is found (don’t pop parentheses) then push
  - Left parentheses - push
  - Right parentheses - pop stack and write to output until left parentheses is found
  - When done processing expression, pop remaining items and write to output
  - NOTE - parentheses are not written to the output

Stack Application - Infix to Postfix Conversion

a + b * c - (d * e + f) * g

Rule	Stack	Output
Operand - write to output		a
	+	a
	+	ab
	+*	ab
	+*	abc
	-	abc*+
	-(	abc*+
	-(	abc*+d
	-(*	abc*+d
.	-(*	abc*+de

Stack Application - Infix to Postfix Conversion

a + b * c - (d * e + f) * g

Rule	Stack	Output
When done processing expression, pop remaining items and write to output	-(+	abc*+de*
	-(+	abc*+de*f
	-	abc*+de*f+
	-*	abc*+de*f+
	-*	abc*+de*f+g
		abc*+de*f+g*-

Stack Application - Evaluating Postfix Expressions

- You may assume I give you a valid postfix expression
- Algorithm
  - Process postfix expression one item at a time
  - Operand - push
  - Operator - pop 2 times, evaluate expression (second_pop operator first_pop), push result onto stack

Stack Application - Evaluating Postfix Expressions

6 * (5 + ((2 + 3) * 8) + 3) => **6 5 2 3 + 8 * + 3 + ***

Current Symbol	Stack
6	6
5	6 5
2	6 5 2
3	6 5 2 3
+	6 5 5

Stack Application - Evaluating Postfix Expressions

6 * (5 + ((2 + 3) * 8) + 3) => **6 5 2 3 + 8 * + 3 + ***

Current Symbol	Stack
8	6 5 5 8
*	6 5 40
+	6 45
3	6 45 3
+	6 48
*	288

Other Stack Applications

- Balanced brace problem
  - Push every left brace
  - When you find a right brace, pop and compare. If no matching left brace then error
  - If stack doesn't end up empty then error
- Path problem
  - Take a path and return in the reverse order

Growable Array-based Stack

- In a push operation, when the array is full, instead of throwing an exception, we can replace the array with a larger one
- How large should the new array be?
  - incremental strategy: increase the size by a constant *c*
  - doubling strategy: double the size

```
Algorithm push(o)
  if t = S.length − 1 then
    A ← new array of
        size …
    for i ← 0 to t do
      A[i] ← S[i]
      S ← A
  t ← t + 1
  S[t] ← o
```

Linked List Based Stack

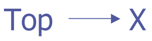
- Using a linked list can remove the size restrictions of an array
- Head will be referred to as the top
- Top initially points to NULL
- All operations and done at the top
  - Push = Insert at head/top
  - Pop = Remove from head/top

Linked List Based Stack

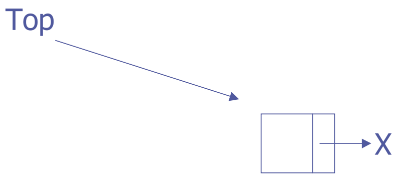
```
bool isEmpty ( ) {
        if ( top == NULL )
                return true;
        else
                return false;
}

void push ( Node* newTop )  {
        newTop->next = top;
        top = newTop;
}
```

```
Node* getTop ( ) {
        return top;
}
```

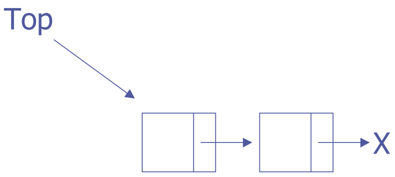
Linked List Based Stack Operations



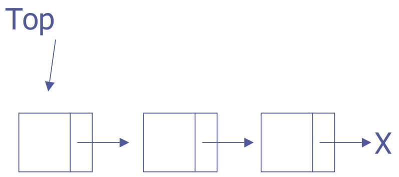
Linked List Based Stack Operations



Linked List Based Stack Operations



Linked List Based Stack Operations



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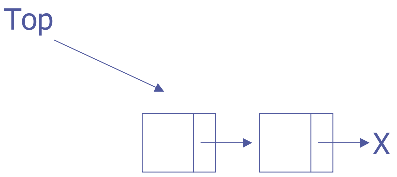

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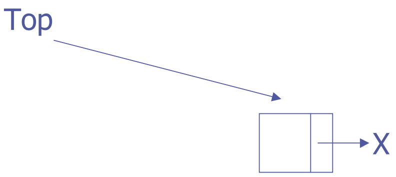

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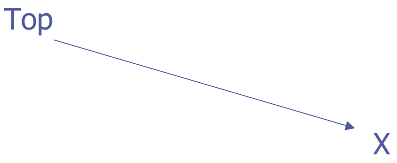

Linked List Based Stack

- In class exercise - Write the pop function
  - Think about memory leaks
    - Just delete the node, don't expect user to
  - Use getTop ( ) if you want to use the node
  - Use pop if you just want to remove the node

Linked List Based Stack

Stacks

- Often the array implementation is used since the stack usually never grows very large even when there is a large number of operations

Stack Big Oh Runtimes

- Array based
  - Push
  - Pop
  - isEmpty
  - getTop
- Linked list based
  - Push
  - Pop
  - isEmpty
  - getTop