# MAS VIMU Matematičke osnove mašinskog učenja Kolokvijum 1

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Ovde rešavamo zadatke sa kolokvijuma 1 MAS VIMU Matematičke osnove mašnskog učenja

## 1.

### (a)

Koristićemo R i **matlib** biblioteku.

```
library(matlib)
a=matrix(c(
  2, 3, 0, 1,
 -1, 1, 2, 0,
  0, 2, 1, 1,
  1,-1,-4, 2,
  3, 0,-2,-1),ncol=4,byrow=T); a
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    2    3    0    1
## [2,]   -1    1    2    0
## [3,]    0    2    1    1
## [4,]    1   -1   -4    2
## [5,]    3    0   -2   -1
```

```
y = matrix(c(-8, 3, -5, -17, 1)); y
```

```
##      [,1]
## [1,]   -8
## [2,]    3
## [3,]   -5
## [4,]  -17
## [5,]    1
```

```
# (a)
gaussianElimination(a)
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    1    0    0   -1
## [2,]    0    1    0    1
## [3,]    0    0    1   -1
## [4,]    0    0    0    0
## [5,]    0    0    0    0
```

```
a0=a[,1:3]; a0                    # Prve tri kolone imaju pivota
```

```
##      [,1] [,2] [,3]
## [1,]    2    3    0
```

```
## [2,]   -1    1    2
## [3,]    0    2    1
## [4,]    1   -1   -4
## [5,]    3    0   -2
```

```r
x0 = matrix(c(-1,1,-1,-1)); x0     # -1 trik
```

```
##      [,1]
## [1,]   -1
## [2,]    1
## [3,]   -1
## [4,]   -1
```

$\dim \operatorname{Im}\Phi = 3$, $\operatorname{Im}\Phi = \operatorname{span}\left(\begin{bmatrix} 2 \\ -1 \\ 0 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 2 \\ -1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ 1 \\ -4 \\ -2 \end{bmatrix}\right)$ $\dim \operatorname{Ker}\Phi = 1$, $\operatorname{Ker}\Phi = \operatorname{span}\left(\begin{bmatrix} -1 \\ 1 \\ -1 \\ -1 \end{bmatrix}\right)$

**(b)**

```r
# (b)
a10=MoorePenrose(a0); a10         # Pseudo inverz nad kolonama baze
```

```
##             [,1]        [,2]        [,3]       [,4]       [,5]
## [1,]  0.02543007 -0.06207928 -0.13462977 -0.2550486  0.3807031
## [2,]  0.22513089  0.06806283  0.21989529  0.1832461 -0.1884817
## [3,] -0.05983545  0.02842184 -0.09498878 -0.3410621  0.1630516
```

```r
x10=a10 %*% y; x10                # Projekcica y na bazu
```

```
##      [,1]
## [1,]    5
## [2,]   -6
## [3,]    7
```

```r
round(a0 %*% x10 - y,digits = 8) # Provera da li y pripada Im \Phi
```

```
##      [,1]
## [1,]    0
## [2,]    0
## [3,]    0
## [4,]    0
## [5,]    0
```

$$y = 5\begin{bmatrix} 2 \\ -1 \\ 0 \\ 1 \\ 3 \end{bmatrix} - 6\begin{bmatrix} 3 \\ 1 \\ 2 \\ -1 \\ 0 \end{bmatrix} + 7\begin{bmatrix} 0 \\ 2 \\ 1 \\ -4 \\ -2 \end{bmatrix}$$

**(c)**

```r
# (c)
x1=rbind(x10,0); x1               # Original nad svim kolonama
```

```
##      [,1]
## [1,]    5
```

```
## [2,]   -6
## [3,]    7
## [4,]    0
```

```r
round(a %*% x1 - y,digits = 8)    # Provera da li je A x_1 = y
```

```
##      [,1]
## [1,]    0
## [2,]    0
## [3,]    0
## [4,]    0
## [5,]    0
```

```r
x2=x1+x0; x2                        # Još jedno rešenje u pravcu vektora Ker \Phi
```

```
##      [,1]
## [1,]    4
## [2,]   -5
## [3,]    6
## [4,]   -1
```

```r
round(a %*% x2 - y,digits = 8)    # Provera
```

```
##      [,1]
## [1,]    0
## [2,]    0
## [3,]    0
## [4,]    0
## [5,]    0
```

$x_1 = \begin{bmatrix} 5 \\ -6 \\ 7 \\ 0 \end{bmatrix}$ Jedinstveno? Ne. $x_2 = \begin{bmatrix} 4 \\ -5 \\ 6 \\ -1 \end{bmatrix}$

### (d)

$x_3$ nekolinearno sa $x_2 - x_1$ ne postoji zato što je $\dim \mathrm{Ker}\Phi = 1$.

## 2.

### (a)

```r
u=matrix(c( 0, -1,  2,  0,  2,
           -3,  4,  1,  2,  1,
            1, -3,  1,- 1,  2,
           -1, -3,  5,  0,  7),ncol=4); u
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    0   -3    1   -1
## [2,]   -1    4   -3   -3
## [3,]    2    1    1    5
## [4,]    0    2   -1    0
## [5,]    2    1    2    7
```

```r
x=matrix(c(-9, -1, -1,  4,  1)); x
```

```
##      [,1]
```

```
## [1,]   -9
## [2,]   -1
## [3,]   -1
## [4,]    4
## [5,]    1
```

```r
# (a)
Px=Proj(x,u); Px
```

```
## [1] -5.095238  2.746032 -2.015873  1.428571  3.888889
```

```r
library(MASS)
print(fractions(Px))
```

```
## [1] -107/21  173/63 -127/63     10/7     35/9
```

```r
Px*63
```

```
## [1] -321  173 -127   90  245
```

$$\pi_U(x) = \frac{1}{63}\begin{bmatrix} -321 \\ 173 \\ -127 \\ 90 \\ 245 \end{bmatrix} = \begin{bmatrix} -5.0952 \\ 2.7460 \\ -2.0159 \\ 1.4286 \\ 3.8889 \end{bmatrix}$$

### (b)

```r
# (b)
xPx=matrix(x-Px,ncol=1)
print(fractions(t(xPx) %*% xPx))
```

```
##       [,1]
## [1,] 2852/63
```

```r
sqrt(t(xPx) %*% xPx)
```

```
##          [,1]
## [1,] 6.728287
```

$d(x, U) = \sqrt{\frac{2852}{63}} = 6.728287$

## 3.

### (a)

```r
a=matrix(c(4, 2, 3, -3, -1, -3, -2, -2, -1),ncol=3); a
```

```
##      [,1] [,2] [,3]
## [1,]    4   -3   -2
## [2,]    2   -1   -2
## [3,]    3   -3   -1
```

```r
u1=matrix(c(1,0,1),ncol=1)
u2=matrix(c(1,1,0),ncol=1)
u3=matrix(c(1,1,1),ncol=1)

# (a)
```

```r
a %*% u1
```

```
##      [,1]
## [1,]    2
## [2,]    0
## [3,]    2
```

```r
a %*% u2
```

```
##      [,1]
## [1,]    1
## [2,]    1
## [3,]    0
```

```r
a %*% u3
```

```
##      [,1]
## [1,]   -1
## [2,]   -1
## [3,]   -1
```

Vidimo da je

$$\begin{bmatrix} 4 & -3 & -2 \\ 2 & -1 & -2 \\ 3 & -3 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix} = 2 \cdot \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix},$$

$$\begin{bmatrix} 4 & -3 & -2 \\ 2 & -1 & -2 \\ 3 & -3 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = 1 \cdot \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix},$$

$$\begin{bmatrix} 4 & -3 & -2 \\ 2 & -1 & -2 \\ 3 & -3 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix} = -1 \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$$

$\lambda_1 = 2, \lambda_2 = 1, \lambda_3 = -1.$

### (b)

```r
# (b)
p=cbind(u1,u2,u3); det(p)
```

```
## [1] 1
```

```r
p1=solve(p); p1
```

```
##      [,1] [,2] [,3]
## [1,]    1   -1    0
## [2,]    1    0   -1
## [3,]   -1    1    1
```

```r
d=diag(c(2,1,-1)); d
```

```
##      [,1] [,2] [,3]
## [1,]    2    0    0
## [2,]    0    1    0
## [3,]    0    0   -1
```

```r
p %*% d %*% p1
```

```
##      [,1] [,2] [,3]
## [1,]    4   -3   -2
## [2,]    2   -1   -2
## [3,]    3   -3   -1
```

```r
zapsmall(p %*% d %*% p1 - a)     # Provera
```

```
##      [,1] [,2] [,3]
## [1,]    0    0    0
## [2,]    0    0    0
## [3,]    0    0    0
```

Dijagonalizabilna? Da. $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ -1 & 1 & 1 \end{bmatrix}$