MML Book Exercises rešavanje

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Ovde rešavamo zadatke iz MML Book

MML-book Chapter 4. Matrix Decompositions, Exercises

Koristićemo R i **matlib** biblioteku.

```r
library(matlib)
```

Tekst je u formatu Rmd (R markdown) i može se knit aplikacijom pretvoriti u html (ili pdf) dokument.

4.1

$$\begin{vmatrix}1 & 3 & 5\\2 & 4 & 6\\0 & 2 & 4\end{vmatrix} = 1\cdot\begin{vmatrix}4 & 6\\2 & 4\end{vmatrix} - 3\cdot\begin{vmatrix}2 & 6\\0 & 4\end{vmatrix} + 5\cdot\begin{vmatrix}2 & 4\\0 & 2\end{vmatrix} = 4\cdot4-2\cdot6-3\cdot(2\cdot4-0\cdot6)+5\cdot(2\cdot2-0\cdot4)=0$$

```r
#########
#  4.1  #
#########

A = matrix(c(1,3,5,2,4,6,0,2,4),ncol = 3, byrow= "T"); det(A)
```

```
## [1] 0
```

4.2

Razvićemo po drugoj koloni: $\begin{vmatrix}2 & 0 & 1 & 2 & 0\\2 & -1 & 0 & 1 & 1\\0 & 1 & 2 & 1 & 2\\-2 & 0 & 2 & -1 & 2\\2 & 0 & 0 & 1 & 1\end{vmatrix} = -1\cdot\begin{vmatrix}2 & 1 & 2 & 0\\0 & 2 & 1 & 2\\-2 & 2 & -1 & 2\\2 & 0 & 1 & 1\end{vmatrix} - 1\cdot\begin{vmatrix}2 & 1 & 2 & 0\\2 & 0 & 1 & 1\\-2 & 2 & -1 & 2\\2 & 0 & 1 & 1\end{vmatrix} = \cdots = 6$

```r
#########
#  4.2  #
#########

A = matrix(c(2,0,1,2,0,2,-1,0,1,1,0,1,2,1,2,-2,0,2,-1,2,2,0,0,1,1),ncol = 5);  det(A)
```

```
## [1] 6
```

4.3

a. $\begin{bmatrix}1 & 0\\1 & 1\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} = \lambda\begin{bmatrix}x_1\\x_2\end{bmatrix} \Leftrightarrow \begin{bmatrix}1-\lambda & 0\\1 & 1-\lambda\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix} \Leftrightarrow \begin{vmatrix}1-\lambda & 0\\1 & 1-\lambda\end{vmatrix} = (1-\lambda)^2 = 0 \Leftrightarrow \lambda_{1,2} = 1$ i $\begin{bmatrix}0 & 0\\1 & 0\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix} \Leftrightarrow x_1 = 0$ što daje jednodimenzionalni prostor koji odgovara $\lambda = 1$: $E_1 = \text{span}([0,1]^T)$.

b. $\begin{bmatrix}-2 & 2\\ 2 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} = \lambda\begin{bmatrix}x_1\\ x_2\end{bmatrix} \Leftrightarrow \begin{bmatrix}-2-\lambda & 2\\ 2 & 1-\lambda\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix} \Leftrightarrow \begin{vmatrix}-2-\lambda & 2\\ 2 & 1-\lambda\end{vmatrix} = \lambda^2+\lambda-6 = 0$ Dobijamo $\lambda = 2 \vee \lambda = -3$. Za $\lambda = 2$: $\begin{bmatrix}-4 & 2\\ 2 & -1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix} \Leftrightarrow x_2 = 2x_1$ što daje jednodimenzionalni prostor $E_2 = \text{span}([1,2]^T)$. Za $\lambda = -3$: $\begin{bmatrix}1 & 2\\ 2 & 4\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix} \Leftrightarrow x_1 = -2x_2$ što daje jednodimenzionalni prostor $E_{-3} = \text{span}([-2,1]^T)$.

```
#########
#  4.3  #
#########

# b
A = matrix(c(-2,2,2,1),ncol = 2, byrow = "T"); A
```

```
##      [,1] [,2]
## [1,]   -2    2
## [2,]    2    1
```

```
eigen(A)->ea; ea
```

```
## eigen() decomposition
## $values
## [1]  2 -3
##
## $vectors
##            [,1]       [,2]
## [1,] -0.4472136 -0.8944272
## [2,] -0.8944272  0.4472136
```

```
round(ea$vectors %*% diag(-c(sqrt(5),-sqrt(5))),digits=8)
```

```
##      [,1] [,2]
## [1,]    1   -2
## [2,]    2    1
```

4.4

```
#########
#  4.4  #
#########

A = matrix(c(0,-1,1,1,-1,1,-2,3,2,-1,0,0,1,-1,1,0),ncol = 4, byrow = "T"); A
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    0   -1    1    1
## [2,]   -1    1   -2    3
## [3,]    2   -1    0    0
## [4,]    1   -1    1    0
```

```
eigen(A)->ea; ea
```

```
## eigen() decomposition
## $values
## [1]  2+0.000000e+00i  1+0.000000e+00i -1+2.720552e-08i -1-2.720552e-08i
##
## $vectors
```

```
##                  [,1]   [,2]                        [,3]
## [1,] 5.773503e-01+0i 0.5+0i -3.042967e-16-1.923721e-08i
## [2,] 2.319703e-16+0i 0.5+0i -7.071068e-01+0.000000e+00i
## [3,] 5.773503e-01+0i 0.5+0i -7.071068e-01+1.923721e-08i
## [4,] 5.773503e-01+0i 0.5+0i  8.012345e-16+8.740850e-24i
##                             [,4]
## [1,] -3.042967e-16+1.923721e-08i
## [2,] -7.071068e-01+0.000000e+00i
## [3,] -7.071068e-01-1.923721e-08i
## [4,]  8.012345e-16-8.740850e-24i
```

```
zapsmall(ea$values)
```

```
## [1]  2+0i  1+0i -1+0i -1+0i
```

```
lambda=Re(ea$values); zapsmall(lambda)
```

```
## [1]  2  1 -1 -1
```

```
vecs=Re(ea$vectors); zapsmall(vecs)
```

```
##           [,1] [,2]       [,3]       [,4]
## [1,] 0.5773503  0.5  0.0000000  0.0000000
## [2,] 0.0000000  0.5 -0.7071068 -0.7071068
## [3,] 0.5773503  0.5 -0.7071068 -0.7071068
## [4,] 0.5773503  0.5  0.0000000  0.0000000
```

```
vecs=vecs %*% diag(c(sqrt(3),2,-sqrt(2),-sqrt(2)))
round(vecs,digits=8)
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    1    1    0    0
## [2,]    0    1    1    1
## [3,]    1    1    1    1
## [4,]    1    1    0    0
```

Vidimo da su karakteristične vrednosti i karakteristični prostori koji im odgovaraju: $\lambda = 2$, $E_2 = \text{span}([1,0,1,1]^T)$, $\lambda = 1$, $E_1 = \text{span}([1,1,1,1]^T)$, $\lambda = -1$ algebarski dvostruki, geometrijski jednostruki: $E_{-1} = \text{span}([0,1,1,0]^T)$.

4.5

- $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}^{-1} = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} = \text{diag}(1,1)$ invertibilna i dijagonalizabilna
- $\det(\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}) = 0$, $\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix} = \text{diag}(1,0)$, nije invertibilna, jeste dijagonalizabilna
- $\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}^{-1} = \begin{bmatrix}1 & -1\\0 & 1\end{bmatrix}$ $\det(\begin{bmatrix}1-\lambda & 1\\0 & 1-\lambda\end{bmatrix}) = 0 \Rightarrow \lambda = 1$ dvostruka nula. Za $\lambda = 1$: $\begin{bmatrix}0 & 1\\0 & 0\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix} \Leftrightarrow x_2 = 0$ što daje jednodimenzionalni prostor $E_1 = \text{span}([1,0]^T)$. Jeste invertibilna, nije dijagonalizabilna
- $\det(\begin{bmatrix}0 & 1\\0 & 0\end{bmatrix}) = 0$, $\det(\begin{bmatrix}-\lambda & 1\\0 & -\lambda\end{bmatrix}) = 0 \Rightarrow \lambda = 0$ dvostruka nula. Za $\lambda = 0$: $\begin{bmatrix}0 & 1\\0 & 0\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix} \Leftrightarrow x_2 = 0$ što daje jednodimenzionalni prostor $E_0 = \text{span}([1,0]^T)$. Nije invertibilna, nije dijagonalizabilna

4.6

```
#########
#  4.6  #
#########

# a

A = matrix(c(2,1,0,3,4,0,0,3,1), ncol = 3); A
```

```
##      [,1] [,2] [,3]
## [1,]    2    3    0
## [2,]    1    4    3
## [3,]    0    0    1
```

```
ev = eigen(A)
ev$values
```

```
## [1] 5 1 1
```

```
ev$vectors
```

```
##            [,1]       [,2]          [,3]
## [1,] -0.7071068 -0.9486833  9.486833e-01
## [2,] -0.7071068  0.3162278 -3.162278e-01
## [3,]  0.0000000  0.0000000  9.362223e-17
```

```
zapsmall(ev$vectors)
```

```
##            [,1]       [,2]       [,3]
## [1,] -0.7071068 -0.9486833  0.9486833
## [2,] -0.7071068  0.3162278 -0.3162278
## [3,]  0.0000000  0.0000000  0.0000000
```

```
round(matrix(ev$vectors,ncol=3) %*% diag(-c(sqrt(2),sqrt(10),-sqrt(10))),digits=8)
```

```
##      [,1] [,2] [,3]
## [1,]    1    3    3
## [2,]    1   -1   -1
## [3,]    0    0    0
```

a. Matrica A nije dijagonazibilna. Sopstveni prostori koji odgovaraju karakterističnim vrednostima su: Za $\lambda = 5$: $E_5 = \text{span}([1,1,0]^T)$ i Za $\lambda = 1$ dvostruki algebarski: jednostruki geometrijski $E_1 = \text{span}([3,-1,0]^T)$

```
#########
#  4.6  #
#########

# b

A = matrix(c(1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0), ncol = 4, byrow = T ); A
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    1    1    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
```

```r
ev=eigen(A)
round(ev$values, digits = 8)
```

```
## [1] 1 0 0 0
```

```r
round(ev$vectors, digits = 8)
```

```
##      [,1]       [,2] [,3] [,4]
## [1,]    1 -0.7071068    0    0
## [2,]    0  0.7071068    0    0
## [3,]    0  0.0000000    1    0
## [4,]    0  0.0000000    0    1
```

```r
evx = round(ev$vectors %*% diag(c(1,-sqrt(2),1,1)), digits = 8); evx
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    1    1    0    0
## [2,]    0   -1    0    0
## [3,]    0    0    1    0
## [4,]    0    0    0    1
```

```r
evx1 = solve(evx); round(evx1, digits = 8 )
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    1    1    0    0
## [2,]    0   -1    0    0
## [3,]    0    0    1    0
## [4,]    0    0    0    1
```

```r
evx %*% evx1
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    1    0    0    0
## [2,]    0    1    0    0
## [3,]    0    0    1    0
## [4,]    0    0    0    1
```

```r
evx1 %*% diag(c(1,0,0,0)) %*% evx
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    1    1    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
```

b. Matrica A je dijagonazibilna. Sopstveni prostori koji odgovaraju karakterističnim vrednostima su: Za $\lambda = 1$: $E_5 = \text{span}([1, 0, 0, 0]^T)$ i Za $\lambda = 0$ trostruki algebarski i trostruki geometrijski: $E_0 = \text{span}([1, -1, 0, 0]^T, [0, 0, 1, 0]^T, [0, 0, 0, 1]^T)$

4.7

```r
#########
#  4.7  #
#########

# a

A = matrix(c(0,-8,1,4), ncol = 2); A
```

```
##      [,1] [,2]
## [1,]    0    1
## [2,]   -8    4
```

```
ev=eigen(A);
round(ev$values, digits = 8)
```

```
## [1] 2+2i 2-2i
```

a. Matrica A nije dijagolizabilna (nad $\mathbb{R}$) jer nema korene nad skupom $\mathbb{R}$.

```
#########
#  4.7  #
#########

# b

A = matrix(c(1,1,1,1,1,1,1,1,1), ncol = 3); A
```

```
##      [,1] [,2] [,3]
## [1,]    1    1    1
## [2,]    1    1    1
## [3,]    1    1    1
```

```
ev=eigen(A)
round(ev$values, digits = 8)
```

```
## [1] 3 0 0
```

```
round(ev$vectors, digits = 8)
```

```
##            [,1]       [,2]       [,3]
## [1,] -0.5773503  0.8164966  0.0000000
## [2,] -0.5773503 -0.4082483 -0.7071068
## [3,] -0.5773503 -0.4082483  0.7071068
```

```
p=round(ev$vectors %*% diag(c(-sqrt(3),sqrt(6),sqrt(2))), digits = 8); p
```

```
##      [,1] [,2] [,3]
## [1,]    1    2    0
## [2,]    1   -1   -1
## [3,]    1   -1    1
```

```
p1=solve(p); p1
```

```
##           [,1]       [,2]       [,3]
## [1,] 0.3333333  0.3333333  0.3333333
## [2,] 0.3333333 -0.1666667 -0.1666667
## [3,] 0.0000000 -0.5000000  0.5000000
```

```
zapsmall(6*p1)
```

```
##      [,1] [,2] [,3]
## [1,]    2    2    2
## [2,]    2   -1   -1
## [3,]    0   -3    3
```

```
p %*% diag(c(3,0,0))%*% p1
```

```
##      [,1] [,2] [,3]
```

```
## [1,]    1    1    1
## [2,]    1    1    1
## [3,]    1    1    1
```

b. Matrica A je dijagolizabilna: $\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix} = \begin{bmatrix}1&2&0\\1&-1&-1\\1&-1&1\end{bmatrix}\begin{bmatrix}3&0&0\\0&0&0\\0&0&0\end{bmatrix}\frac{1}{6}\begin{bmatrix}2&2&2\\2&-1&-1\\0&-3&3\end{bmatrix}$, gde je $\frac{1}{6}\begin{bmatrix}2&2&2\\2&-1&-1\\0&-3&3\end{bmatrix} = \begin{bmatrix}1&2&0\\1&-1&-1\\1&-1&1\end{bmatrix}^{-1}$.

```
#########
#  4.7  #
#########

# c

A = matrix(c(5,0,-1,1,4,1,-1,1,2,-1,3,-1,1,-1,0,2), ncol = 4); A
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    5    4    2    1
## [2,]    0    1   -1   -1
## [3,]   -1   -1    3    0
## [4,]    1    1   -1    2
```

```
ev = eigen(A)
ev$values
```

```
## [1] 4 4 2 1
```

```
ev$vectors
```

```
##            [,1]       [,2]          [,3]          [,4]
## [1,] -0.5773503  0.5773503 -5.773503e-01 -7.071068e-01
## [2,]  0.0000000  0.0000000  5.773503e-01  7.071068e-01
## [3,]  0.5773503 -0.5773503  4.079220e-16  1.480385e-16
## [4,] -0.5773503  0.5773503 -5.773503e-01  4.261214e-16
```

```
zapsmall(ev$vectors)
```

```
##            [,1]       [,2]       [,3]       [,4]
## [1,] -0.5773503  0.5773503 -0.5773503 -0.7071068
## [2,]  0.0000000  0.0000000  0.5773503  0.7071068
## [3,]  0.5773503 -0.5773503  0.0000000  0.0000000
## [4,] -0.5773503  0.5773503 -0.5773503  0.0000000
```

```
round(matrix(ev$vectors,ncol=4) %*% diag(c(-sqrt(3),sqrt(3),-sqrt(3),-sqrt(2))),digits=8)
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    1    1    1    1
## [2,]    0    0   -1   -1
## [3,]   -1   -1    0    0
## [4,]    1    1    1    0
```

c. Matrica A nije dijagonazibilna. Sopstveni prostori koji odgovaraju karakterističnim vrednostima su: Za $\lambda = 4$ dvostruki algebarski: jednostruki geometrijski $E_4 = \text{span}([1,0,-1,1]^T)$, za $\lambda = 2$: $E_2 = \text{span}([1,1,-1,0,1]^T)$ i za $\lambda = 1$: $E_1 = \text{span}([1,-1,0,0]^T)$.

```r
#########
#  4.7  #
#########

# d

A = matrix(c(5,-1,3,-6,4,-6,-6,2,-4), ncol = 3); A
```

```
##      [,1] [,2] [,3]
## [1,]    5   -6   -6
## [2,]   -1    4    2
## [3,]    3   -6   -4
```

```r
ev=eigen(A)
round(ev$values, digits = 8)
```

```
## [1] 2 2 1
```

```r
round(ev$vectors, digits = 8)
```

```
##            [,1]       [,2]       [,3]
## [1,] -0.6854346 -0.9414664 -0.6882472
## [2,]  0.3141575 -0.1976432  0.2294157
## [3,] -0.6568748 -0.2730900 -0.6882472
```

```r
p=round(ev$vectors, digits = 8); p
```

```
##            [,1]       [,2]       [,3]
## [1,] -0.6854346 -0.9414664 -0.6882472
## [2,]  0.3141575 -0.1976432  0.2294157
## [3,] -0.6568748 -0.2730900 -0.6882472
```

```r
p1=solve(p); p1
```

```
##           [,1]       [,2]      [,3]
## [1,] -4.016414  9.2993609  7.116201
## [2,] -1.324541 -0.3973624  1.192087
## [3,]  4.358899 -8.7177980 -8.717798
```

```r
round(p %*% diag(c(2,2,1))%*% p1,digits=8)
```

```
##      [,1] [,2] [,3]
## [1,]    5   -6   -6
## [2,]   -1    4    2
## [3,]    3   -6   -4
```

d. Matrica A je dijagolizabilna.

4.8

```r
#########
#  4.8  #
#########

A = matrix(c(3,2,2,3,2,-2), ncol = 3); A
```

```
##      [,1] [,2] [,3]
## [1,]    3    2    2
```

```
## [2,]    2    3   -2
```

```r
svda = svd(A, nu=2, nv = 3); svda
```

```
## $d
## [1] 5 3
##
## $u
##            [,1]       [,2]
## [1,] -0.7071068 -0.7071068
## [2,] -0.7071068  0.7071068
##
## $v
##               [,1]       [,2]       [,3]
## [1,] -7.071068e-01 -0.2357023 -0.6666667
## [2,] -7.071068e-01  0.2357023  0.6666667
## [3,]  1.604701e-16 -0.9428090  0.3333333
```

```r
U = svda$u
S = 0*A
for(k in 1:min(dim(A))) {S[k,k] = svda$d[k]}
V = svda$v
U%*% S %*% t(V)
```

```
##      [,1] [,2] [,3]
## [1,]    3    2    2
## [2,]    2    3   -2
```

```r
zapsmall(V)
```

```
##            [,1]       [,2]       [,3]
## [1,] -0.7071068 -0.2357023 -0.6666667
## [2,] -0.7071068  0.2357023  0.6666667
## [3,]  0.0000000 -0.9428090  0.3333333
```

$$\begin{bmatrix} 2 & 3 & -2 \\ 3 & 2 & 2 \end{bmatrix} = \begin{bmatrix} -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} \begin{bmatrix} 5 & 0 & 0 \\ 0 & 3 & 0 \end{bmatrix} \begin{bmatrix} -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{6} & -\frac{2}{3} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{6} & \frac{2}{3} \\ 0 & -\frac{2\sqrt{2}}{3} & \frac{1}{3} \end{bmatrix}^T$$

4.10

```r
#########
#  4.10 #
#########

U[,1] %*% t(V[,1])
```

```
##      [,1] [,2]          [,3]
## [1,]  0.5  0.5 -1.134695e-16
## [2,]  0.5  0.5 -1.134695e-16
```

```r
zapsmall(5 * U[,1] %*% t(V[,1]))
```

```
##      [,1] [,2] [,3]
## [1,]  2.5  2.5    0
## [2,]  2.5  2.5    0
```

$$A_1 = u_1 v_1^T = 5 \begin{bmatrix} -\frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} \end{bmatrix} \begin{bmatrix} -\frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} \\ 0 \end{bmatrix}^T = \begin{bmatrix} \frac{5}{2} & \frac{5}{2} & 0 \\ \frac{5}{2} & \frac{5}{2} & 0 \end{bmatrix} = \begin{bmatrix} 2.5 & 2.5 & 0 \\ 2.5 & 2.5 & 0 \end{bmatrix}$$

4.9

```
#########
# 4.9 #
#########

A = matrix(c(2,-1,2,1), ncol = 2); A
```

```
##      [,1] [,2]
## [1,]    2    2
## [2,]   -1    1
```

```
svda = svd(A, nu=2, nv = 2); svda
```

```
## $d
## [1] 2.828427 1.414214
##
## $u
##                 [,1]         [,2]
## [1,] -1.000000e+00 1.110223e-16
## [2,]  1.110223e-16 1.000000e+00
##
## $v
##            [,1]       [,2]
## [1,] -0.7071068 -0.7071068
## [2,] -0.7071068  0.7071068
```

```
U = svda$u
S = 0*A
for(k in 1:min(dim(A))) {S[k,k] = svda$d[k]}
V = svda$v
U%*% S %*% t(V)
```

```
##      [,1] [,2]
## [1,]    2    2
## [2,]   -1    1
```

```
zapsmall(U)
```

```
##      [,1] [,2]
## [1,]   -1    0
## [2,]    0    1
```

$$\begin{bmatrix} 2 & 2 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2\sqrt{2} & 0 \\ 0 & \sqrt{2} \end{bmatrix} \begin{bmatrix} -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}^T$$