

# Operaciona istraživanja, kolokvijum 1

25. IV 2016. godine

## Teorija

(a)

$$\begin{aligned} \zeta &= x_1 + x_2 && \rightarrow \max \\ x_1 + 5x_2 + w_1 &= 5 \\ 2x_1 + x_2 + w_2 &= 4 \\ x_1 \geq 0 \quad x_2 \geq 0 \quad w_1 \geq 0 \quad w_2 \geq 0 \end{aligned}$$

(b)

$$\begin{aligned} \xi &= 5y_1 + 4y_2 && \rightarrow \min \\ y_1 + 2y_2 &\geq 1 \\ 5y_1 + y_2 &\geq 1 \\ y_1 \geq 0 \quad y_2 \geq 0 \end{aligned}$$

(c)

$$\begin{aligned} -\xi &= -5y_1 - 4y_2 && \rightarrow \max \\ -y_1 - 2y_2 &\leq -1 \\ -5y_1 - y_2 &\leq -1 \\ y_1 \geq 0 \quad y_2 \geq 0 \end{aligned}$$

(d)

$$\begin{aligned} -\xi &= -5y_1 - 4y_2 && \rightarrow \max \\ -y_1 - 2y_2 + z_1 &= -1 \\ -5y_1 - y_2 + z_2 &= -1 \\ y_1 \geq 0 \quad y_2 \geq 0 \quad z_1 \geq 0 \quad z_2 \geq 0 \end{aligned}$$

(e)

$$\begin{array}{r} \zeta = 0 + x_1 + x_2 \\ \hline w_1 = 5 - x_1 - 5x_2 \\ w_2 = 4 - 2x_1 - x_2 \end{array}$$

(f)

$$\begin{array}{r} -\xi = 0 - 5y_1 - 4y_2 \\ \hline z_1 = -1 + y_1 + 2y_2 \\ z_2 = -1 + 5y_1 + y_2 \end{array}$$

(g)

$$\begin{array}{r} \zeta = 2 + 1/2x_2 - 1/2w_2 \\ \hline w_1 = 3 - 9/2x_2 + 1/2w_2 \\ x_1 = 2 - 1/2x_2 - 1/2w_2 \end{array}$$

## Zadatak

Uvešćemo veličine  $x_i =$  broj dana angažovanja  $i$ -tog rudnika,  $i = 1, 2, 3$ ,  $\zeta =$  ukupna cena.

$$\begin{aligned} \zeta &= 18x_1 + 20x_2 + 15x_3 \rightarrow \min \\ 4x_1 + 6x_2 + x_3 &\geq 50 \\ 4x_1 + 4x_2 + 6x_3 &\geq 60 \\ x_1 &\leq 5 \\ x_2 &\leq 5 \\ x_3 &\leq 5 \\ x_1 \geq 0 \quad x_2 \geq 0 \quad x_3 \geq 0 \end{aligned}$$

0	$x_1$	$x_2$	$x_3$	$w_1$	$w_2$	$w_3$	$w_4$	$w_5$	
$w_1$	-4	-6	-1	1	0	0	0	0	-50
$w_2$	-4	-4	-6	0	1	0	0	0	-60
$w_3$	1	0	0	0	0	1	0	0	5
$w_4$	0	1	0	0	0	0	1	0	5
$w_5$	0	0	1	0	0	0	0	1	5
	18	20	15	0	0	0	0	0	0

1	$x_1$	$x_2$	$x_3$	$w_1$	$w_2$	$w_3$	$w_4$	$w_5$	
$w_1$	-10/3	-16/3	0	1	-1/6	0	0	0	-40
$x_3$	2/3	2/3	1	0	-1/6	0	0	0	10
$w_3$	1	0	0	0	0	1	0	0	5
$w_4$	0	1	0	0	0	0	1	0	5
$w_5$	-2/3	-2/3	0	0	1/6	0	0	1	-5
	8	10	0	0	5/2	0	0	0	-150

2	$x_1$	$x_2$	$x_3$	$w_1$	$w_2$	$w_3$	$w_4$	$w_5$	
$x_2$	5/8	1	0	-3/16	1/32	0	0	0	15/2
$x_3$	1/4	0	1	1/8	-3/16	0	0	0	5
$w_3$	1	0	0	0	0	1	0	0	5
$w_4$	-5/8	0	0	3/16	-1/32	0	1	0	-5/2
$w_5$	-1/4	0	0	-1/8	3/16	0	0	1	0
	7/4	0	0	15/8	35/16	0	0	0	-225

3	$x_1$	$x_2$	$x_3$	$w_1$	$w_2$	$w_3$	$w_4$	$w_5$	
$x_2$	0	1	0	0	0	0	1	0	5
$x_3$	0	0	1	1/5	-1/5	0	2/5	0	4
$w_3$	0	0	0	3/10	-1/20	1	8/5	0	1
$x_1$	1	0	0	-3/10	1/20	0	-8/5	0	4
$w_5$	0	0	0	-1/5	1/5	0	-2/5	1	1
	0	0	0	12/5	21/10	0	14/5	0	-232

Optimalna tabela:  $x_1 = 4$ ,  $x_2 = 5$ ,  $x_3 = 4$ ,  $\zeta = 232$ .

Rešavamo  $t$  iz sistema nejednačina:

$$z_N^* + t\Delta z_N \geq 0 \Leftrightarrow [12/5, 21/10, 14/5]^T + t[1/5, -1/5, 2/5]^T \geq [0, 0, 0]^T \Leftrightarrow t \in [-7, 21/2].$$

Cena  $-c_3 = -15$  može da pripada  $[-15 + (-7), -15 + 10.5]$ , odnosno,  $c_3 \in [4.5, 22]$ .